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Abstract. We compare a tax with thresholds ('prices'), and tradable permits ('quantities'), as mechanisms to control total 'emissions' (or other inputs or outputs) from heterogeneous parties with uncertainties in emissions, costs and benefits. The advantage of prices over quantities is much smaller than in Weitzman's (1974) non-tradable model. Steeper marginal benefits no longer necessarily prefer quantities; and under tradable permits, marginal cost uncertainty is an inherent benefit. For global greenhouse gas abatement by 18 regions in 2020, a tax dominates, but by much less than suggested by single-party models, especially when targets are indexed to activity levels. Provided handouts of thresholds and permits are limited, including tax interactions moderately raises the advantages of a tax, and of indexation.

JEL codes: D810, H230, Q580
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1. INTRODUCTION

Formal comparisons of economic control and planning mechanisms under uncertainty date back to a seminal paper by Weitzman (1974). By assuming locally quadratic functions for control costs and benefits with uncorrelated uncertainties for producing or polluting parties in a risk-neutral world, he showed that ‘prices’ (a tax) outperform ‘quantities’ (permits, quotas, licences or regulatory standards) when the marginal cost curve is steeper than the marginal benefit curve at the expected social optimum; and that the degree of outperformance rises as marginal benefits get flatter. Subsequent authors have explored many other aspects of market mechanisms, such as price-quantity hybrids (Roberts and Spence 1976, Pizer 2002), more on correlation of benefit and cost uncertainty (Stavins 1996), and stock effects (Hoel and Karp 2002, Newell and Pizer 2003).

However, the literature has still not accurately covered the most policy-relevant application of Weitzman’s theory, which is the comparison of a tax with *tradable* permits or quotas for controlling externalities under uncertainty.[1] We provide this here for the first time, and include uncertainties in levels as well as control costs, and indexation of targets. The main resulting contributions to the prices vs. quantities literature are that the advantage of prices over quantities is much lower than previously thought; under quantities, marginal cost uncertainty is an inherent benefit, compared to certainty; and steeper marginal benefits no longer necessarily prefer control by quantities.

Our claim of novelty, in such a well-worked field that includes the world’s economically largest future pollution problem, that of controlling

1. Hereafter we use ‘permit’ to mean either permit or quota.

greenhouse gas emissions, is surprising but true. An accurate model of permit trading under uncertainty must include three elements. There must be many parties, to trade in permits; they must have heterogeneous marginal control costs, to motivate trading; and permits must be tradable. In the many-party treatments in Weitzman's Section V and Yohe (1976, Section IV), permits are *non*-tradable (though different from conventional regulatory standards), which creates the obvious inefficiency that marginal control costs are equalised only with negligible probability. Permit trading removes this inefficiency, so his many-party advantage of prices versus quantities will be a significant overestimate for a tax versus tradable permits.

Subsequent literature on prices versus quantities under uncertainty has not filled this gap. Weitzman (1978) also considered many parties, and his benefit function was quite general, but again he had no permit trading. The other papers cited above, and Laffont (1977), Yohe (1978), Pizer (1999), Baldursson and von der Fehr (2004), Quirion (2004), Kelly (2005) and Newell and Pizer (2006), all considered only a single or representative party, where permit trading cannot exist. Newell et al (2005) had many, heterogeneous parties, but focused on permit banking (intertemporal trading) rather than intratemporal permit trading among parties. Finally, Quirion (2005) and Sue Wing et al (forthcoming) focused on emissions uncertainty rather than marginal cost uncertainty, and hence on permit price uncertainty caused by fixed permit targets, but again these were single-party analyses.

We extend Weitzman's (1974) static, partial equilibrium model to include permit trading; and unlike in his Section V, this proves not to be a straightforward extension of the single-party case. We make one key restriction, though. In our model the public (non-market) benefit or cost to each party depends only on the *total* over all parties of some good or bad they each produce (such as polluting emissions), but not on any other aspect of good or bad production, such as its variability or geographical distribution. Apart from this, our model is as broadly applicable as Weitzman's original. So for example, it could be applied to households' commodity demands where there is a short-run cap on total supply, as with water in a drought, to firms' R&D spending levels where public knowledge depends on total spending, or approximately to incompletely mixed emissions, like sulphur dioxide. However, for ease of reading we avoid the abstract language of 'controlling quantities' and write instead of 'abating emissions', naming our model Mechanisms to Abate Total Emissions under Stochasticity, or MATES for short.

As in Weitzman, we ignore strategic interactions among emitting parties, and assume instead that targets and the tax rate are set cooperatively to maximise total expected net benefit. However, we include further generalisations as well as permit trading. First, we allow for three direct uncertainties in each party's emissions, as well as in its marginal abatement costs and benefits. Second, to include a richer policy menu, we allow a flexible degree of indexation between a party's permit allocation (or threshold used with a tax, as in Pezzey 2003), and its activity level (such as GDP for countries, or physical output for firms). A final extension is that we compute tax interaction ('double dividend') effects, as studied for example by Goulder et al (1997, 1999) and (for the uncertainty case) by Quirion (2004), for our heterogeneous-parties, multi-uncertainty setting.

The paper is organised as follows. Section 2 defines the model, and gives theoretical results for total, expected total net benefit when this is what is maximised under risk neutrality. Section 3 applies the model empirically to global greenhouse gas abatement in 2020 under risk neutrality and risk aversion. Section 4 extends both theoretical and empirical results under risk-neutral to include tax-interaction effects, and Section 5 concludes.

2. MODELLING TOTAL EMISSIONS ABATEMENT UNDER RISK NEUTRALITY

2.1 Parties, emissions and abatement

In our model of Mechanisms to Abate Total Emissions under Stochasticity (MATES) there are n unevenly-sized parties, either countries or firms within a country, indexed by subscript i (or sometimes k) = 1,...,n. However, the uneven sizing means that n itself plays no role in the model. Each party discharges emissions into a common environment where emissions are completely mixed. Emission abatement is achieved by a market mechanism, denoted by superscript j. The same mechanism is applied to all parties by an 'authority', a (perfectly enforced) cooperative treaty where parties are countries, or a (perfectly enforced) national law where parties are firms. Emission and abatement levels are then denoted:

$\tilde{E}^b_i$ = party i's projected, uncertain, business-as-usual (BAU, denoted ^{b}) emissions, realised at some future time

$\tilde{Q}^j_i$ = i's uncertain, realised amount of abatement in future, induced by abatement mechanism j.

$\tilde{E}^j_i$ = i's uncertain, realised, abated emissions under mechanism j:

$\tilde{E}^j_i := \tilde{E}^b_i - \tilde{Q}^j_i$, hence $\tilde{Q}^j_i = \tilde{E}^b_i - \tilde{E}^j_i$, in units of say tonnes per year (t/yr). [2.1]

All underlying parameters are assumed to be known to all parties. Quantities with tilde overscripts ($\tilde{\ }$) are random variables, and dropping the tilde denotes expectation, so with J being any variable, $J := \mathrm{E}[\tilde{J}]$. Dropping a subscript denotes summation, so $J := \Sigma^n_{i=1} J_i \equiv \Sigma J_i \equiv \Sigma J_k$; and curly brackets denote an n-vector: $\{J_i\} := (J_1,...,J_n)$.

2.2 Mechanisms for abating total emissions

The abatement mechanism can be either allocated targets of tradable emission permits (emissions trading, or control by 'quantities', denoted by $j = T$); or an emission tax with targets that function as thresholds (control by 'price', denoted by $j = \$$). (Non-tradable permits, denoted $j = @$, are considered in Section 2.8.) Each mechanism comprises a set of uncertain target levels $\{\tilde{X}^j_i\}$, plus a certain tax rate $p^\$$ in the case of the tax. Target levels are uncertain or 'flexible' when indexed to activity levels, as explained in Section 2.3.2. For simplicity in our basic model, we assume that all permits or tax thresholds are given away free, so that (with some adjustment, for a tax), each mechanism is revenue-neutral overall; but in Section 4, we will allow the proportion of free permits or thresholds to range between 0 and 1.

2.2.1 Tradable permits (^{T})

Each target (allocation) of emission permits, $\{\tilde{X}^T_i\}$, is tradable, so that perfect enforcement (which is assumed) makes abated emissions equal target emissions only in total:

$$\tilde{E}^T = \tilde{X}^T \text{ t/yr. (with } X^T \text{ chosen} < E^b). \qquad [2.2]$$

Permit trading results in a (positive) common unit price, denoted $\tilde{p}^T$ \$/t, which all parties take as given (we ignore any market power). A party's (realised) trading revenue (often negative), denoted $\tilde{R}^T_i$, is then its permit target $\tilde{X}^T_i$ minus its abated emissions $\tilde{E}^T_i$, times price $\tilde{p}^T$. With [2.1] this gives:

$$\tilde{R}^T_i := \tilde{p}^T[\tilde{X}^T_i - \tilde{E}^b_i + \tilde{Q}^T_i(\tilde{p}^T)], \quad \text{in say \$/yr,} \qquad [2.3]$$

and total revenue-neutrality is automatic ($\tilde{R}^T \equiv 0$ from [2.2] and [2.1]).

2.2.2 *A tax with thresholds* ($^\$$)

To create this mechanism, the authority defines targets $\{\tilde{X}^\$_i\}$ (with $X^\$ < E^b$), which also function as tax thresholds, and chooses a common, certain, positive tax $p^\$$ on each unit of emissions.[2] Each party's realised, gross, positive or negative tax payment is then $p^\$(\tilde{E}^\$_i - \tilde{X}^\$_i)$, which means it receives revenue *from* the authority of $p^\$[\tilde{X}^\$_i - \tilde{E}^b_i + \tilde{Q}^\$_i(p^\$)]$, analogous to trading revenue in [2.3]. Thresholds are treated as quasi-property rights like tradable permits, and so have the same long-run efficiency properties (Pezzey 1992, 2003). The authority chooses $p^\$$ so that *expected* total abated emissions equal expected total target emissions:

$$E^\$(p^\$) = E^b - Q^\$(p^\$) = X^\$ \quad \text{t/yr.} \qquad [2.4]$$

However, the authority's *realised* total payment is generally not zero, so it also promises to claw back a small, lump-sum (positive or negative) amount of $(X^\$_i/X^\$)p^\$[\tilde{X}^\$ - \tilde{E}^b + \tilde{Q}^\$(p^\$)]$ from party i, which then gets a net payment of

$$\tilde{R}^\$_i := p^\$[\tilde{X}^\$_i - \tilde{E}^b_i + \tilde{Q}^\$_i(p^\$)] - (X^\$_i/X^\$)p^\$[\tilde{X}^\$ - \tilde{E}^b + \tilde{Q}^\$(p^\$)] \quad \$/\text{yr}, \qquad [2.5]$$

giving total revenue-neutrality again ($\tilde{R}^\$ \equiv 0$).

2.3 Uncertainties in emissions, and indexation of targets

2.3.1 *Three uncertainties in business-as-usual emission levels*

We consider three random variables as sources of uncertainty in party i's BAU emissions $\tilde{E}^b_i$:

- ε_{Yi}, the proportional random variation in party i's overall *activity level*, denoted $\tilde{Y}_i$ (such as product output for a firm, or GDP for a country), to

2. Because the price is certain, it turns out that the flexibility of tax thresholds is advantageous only under risk-aversion, modelled numerically in Section 3.2.

which a fraction α_i of E^b_i, its expected BAU emissions, is linked;

- $\varepsilon_{\eta i}$, representing proportional uncertainty in party i's *intensity* (emissions/activity ratio), denoted $\tilde{\eta}_i$;
- $\varepsilon_{\rho i}$, representing proportional uncertainty in the other, $(1-\alpha_i)$ fraction of expected BAU emissions which have no link to the activity level.

The net effect of these three uncertainties is that i's BAU emissions are assumed to be

$$\tilde{E}^b_i \quad \approx [1+\alpha_i(\varepsilon_{Yi}+\varepsilon_{\eta i})+(1-\alpha_i)\varepsilon_{\rho i}]\ E^b_i \quad \text{t/yr.} \qquad [2.6]$$

We assume all errors are independent with zero means:

$$\mathrm{E}[\varepsilon_{Yi}] = \mathrm{E}[\varepsilon_{\eta i}] = \mathrm{E}[\varepsilon_{\rho i}] = \mathrm{E}[\varepsilon_{Yi}\varepsilon_{\eta k}] = \mathrm{E}[\varepsilon_{\eta i}\varepsilon_{\rho k}], \text{ etc} = 0 \quad \forall\ i,k \qquad [2.7]$$

$$\mathrm{E}[\varepsilon_{Yi}^2] =: \sigma_{Yi}^2;\ \ \mathrm{E}[\varepsilon_{\eta i}^2] =: \sigma_{\eta i}^2;\ \ \mathrm{E}[\varepsilon_{\rho i}^2] =: \sigma_{\rho i}^2 \quad \forall\ i,k \qquad [2.8]$$

For the case of greenhouse gas (GHG) emissions, Jotzo and Pezzey (forthcoming) explain why these assumptions fit well, and they give calibrations for the variance parameters.

2.3.2 Indexation of emission targets

We define a general, flexibly indexed emissions target as

$$\tilde{X}_i := X_i(1+\beta_i\varepsilon_{Yi}),\ \ \beta_i \geq 0,\ \ \text{with } x_i := X_i/E^b_i \ \Rightarrow\ \tilde{X}_i = x_iE^b_i(1+\beta_i\varepsilon_{Yi}). \ [2.9]$$

Here X_i is the expected target level, β_i (not just 0 or 1) is the degree by which i's realised target $\tilde{X}_i$ is indexed to its activity level $\tilde{Y}_i$ (and hence becomes an intensity target), and x_i is the expected target as a proportion of BAU emissions. For brevity we sometimes call the set $\{x_i\}$ the 'target distribution', omitting the 'expected, proportional' qualifier. We assume the authority sets and enforces collectively optimal targets, thus preventing free-riding and other strategic interactions among emitters.

From [2.6] and [2.9], an expression needed for later working is

$$\tilde{E}^b{}_i - \tilde{X}_i = E^b{}_i - X_i + \tilde{N}_{Ei}, \quad \text{where} \qquad [2.10]$$

$$\tilde{N}_{Ei} \quad := [(\alpha_i - \beta_i x_i)\varepsilon_{Yi} + \alpha_i \varepsilon_{\eta i} + (1-\alpha_i)\varepsilon_{\rho i}]\ E^b{}_i \quad \text{t/yr} \qquad [2.11]$$

is i's net uncertainty from both BAU emissions and a flexibly indexed emissions target. From the independence and variance assumptions in [2.7] and [2.8], we have the following result and notation D_{Ei} for i's expected net emissions uncertainty:

$$\mathrm{E}[\tilde{N}_{Ei}{}^2] = [(\alpha_i - \beta_i x_i)^2 \sigma_{Yi}{}^2 + \alpha_i{}^2 \sigma_{\eta i}{}^2 + (1-\alpha_i)^2 \sigma_{\rho i}{}^2]\ (E^b{}_i)^2 =: D_{Ei} \quad (\text{t/yr})^2. \qquad [2.12]$$

2.4 Benefits and costs of abatement, and their uncertainties

Each party incurs a private cost to abate its own emissions by $\tilde{Q}^j{}_i$, and thanks to complete mixing, gets a private benefit from just the *total* level of emissions abatement, $\tilde{Q}^j$. Party i's benefit from total abatement is quadratic:

$$\tilde{B}^j{}_i(\tilde{Q}^j) \quad := (V_i + \varepsilon_{Bi})\tilde{Q}^j - \tfrac{1}{2} W_i (\tilde{Q}^j)^2; \quad V_i \ \ \$/\text{t} > 0, \ \ W_i \ \ \$.\text{yr/t}^2 > 0; \qquad [2.13]$$

$$\mathrm{E}[\varepsilon_{Bi}] = \mathrm{E}[\varepsilon_{Bi}\varepsilon_{Bk}] = 0; \ \ \mathrm{E}[\varepsilon_{Bi}{}^2] =: \sigma_{Bi}{}^2. \qquad [2.14]$$

as is its abatement cost (net of emissions trading or tax revenue $\tilde{R}^j{}_i$):

$$\tilde{C}^j{}_i(\tilde{Q}^j{}_i) \quad := \tfrac{1}{2}(1/M_i)(\tilde{Q}^j{}_i)^2 + \tilde{Q}^j{}_i \varepsilon_{Ci} - \tilde{R}^j{}_i \ \$/\text{yr}\ ; \qquad [2.15]$$

$$\mathrm{E}[\varepsilon_{Ci}] = \mathrm{E}[\varepsilon_{Ci}\varepsilon_{Ck}] = 0; \ \ \mathrm{E}[\varepsilon_{Ci}{}^2] =: \sigma_{Ci}{}^2 \ \ (\$/\text{t})^2; \ \ \text{and} \qquad [2.16]$$

$$\mathrm{E}[\varepsilon_{Ci}\varepsilon_{Bk}] =: \sigma_{CiBk}{}^2 \ \ (\$/\text{t})^2. \qquad [2.17]$$

For easier presentation later, we define uncertainties scaled by total and per party 'abatement potentials' M and M_i t^2/\$.yr to be

$$\tilde{N}_{Bi} := M\varepsilon_{Bi} \ \ \text{t/yr} \ \ \text{and} \ \ \tilde{N}_{Ci} := M_i \varepsilon_{Ci} \ \ \text{t/yr}. \qquad [2.18]$$

Expected, scaled cost and benefit uncertainties are then

$$\mathrm{E}[\tilde{N}_{Ci}{}^2] = M_i^2\sigma_{Ci}{}^2 =: D_{Ci} \quad \text{and} \quad \mathrm{E}[\tilde{N}_C{}^2] = \Sigma M_i^2\sigma_{Ci}{}^2 = D_C \quad (\text{t/yr})^2; \qquad [2.19]$$

$$\mathrm{E}[\tilde{N}_{Ck}\tilde{N}_{Bi}] = M_k M\sigma_{CkBi}{}^2 =: D_{CkBi}; \;\; \mathrm{E}[\tilde{N}_C\tilde{N}_{Bi}] = \Sigma_k M_k M\sigma_{CkBi}{}^2 = D_{CBi}, \quad (\text{t/yr})^2. \qquad [2.20]$$

and the total marginal abatement benefit and party-level marginal abatement cost curves are respectively

$$\tilde{B}^{j\prime}(\tilde{Q}^j) \;\; = V + \varepsilon_B - W\tilde{Q}^j = V + (1/M)\tilde{N}_B - W\tilde{Q}^j \quad \$/\text{t}, \;\; \text{and} \qquad [2.21]$$

$$\tilde{C}^j{}_i{}'(\tilde{Q}^j{}_i) \;\; = (1/M_i)\tilde{Q}^j{}_i + \varepsilon_{Ci} = (1/M_i)\tilde{Q}^j{}_i + (1/M_i)\tilde{N}_{Ci} \quad \$/\text{t}. \qquad [2.22]$$

Party i's realised net benefit ('advantage') from mechanism j is denoted $\tilde{A}^j{}_i$ $:= \tilde{B}^j{}_i - \tilde{C}^j{}_i$, which from [2.13], [2.15] and [2.18] is

$$\tilde{A}^j{}_i := (V_i+\tilde{N}_{Bi}/M)\tilde{Q}^j - \tfrac{1}{2}W_i(\tilde{Q}^j)^2 - \tfrac{1}{2}(1/M_i)(\tilde{Q}^j{}_i)^2 - (1/M_i)\tilde{Q}^j{}_i\tilde{N}_{Ci} + \tilde{R}^j{}_i \quad \$/\text{yr} \qquad [2.23]$$

2.5 Equating marginal abatement costs with an emissions price

For either tradable permits or a tax, each party maximises its realised net benefit $\tilde{A}^j{}_i$ under the zero conjecture that it cannot affect total abatement, that is, $\partial\tilde{Q}^j/\partial\tilde{Q}^j{}_i = 0$. Party i then chooses its abatement $\tilde{Q}^j{}_i$ to equate its marginal abatement cost $\tilde{C}^j{}_i{}'(\tilde{Q}^j{}_i)$ in [2.22] to the common permit price $\tilde{p}^T$ or tax $p^\$$. For tradable permits, in contrast to the divergence of marginal costs for non-tradable permits noted by Weitzman (1974, p489), this means

$$(1/M_i)\tilde{Q}^T{}_i + (1/M_i)\tilde{N}_{Ci} = \tilde{p}^T \quad \$/\text{t} \;\; \forall i, \;\; \text{hence}$$

$$\tilde{Q}^T{}_i = \tilde{p}^T M_i - \tilde{N}_{Ci}, \;\; \tilde{Q}^T = \tilde{p}^T M - \tilde{N}_C \;\; \text{and} \;\; p^T = Q^T/M. \qquad [2.24]$$

From [2.1], [2.2] and [2.10], total abatement $\tilde{Q}^T$ is also:

$$\tilde{Q}^T = E^b - X^T + \tilde{N}_E, \qquad [2.25]$$

which with [2.24] means the expected permit price is

$$p^T = (E^b - X^T)/M \quad (> 0 \text{ from [2.2]}). \qquad [2.26]^3$$

For a tax, equating marginal abatement cost to the tax rate $p^\$$ means

$$(1/M_i)\tilde{Q}^\$_i + (1/M_i)\tilde{N}_{Ci} = p^\$ \quad \$/\text{t} \quad \forall i, \ \text{hence}$$

$$\tilde{Q}^\$_i = p^\$ M_i - \tilde{N}_{Ci}, \ \ \tilde{Q}^\$ = p^\$ M - \tilde{N}_C, \ \text{and} \ \ p^\$ = Q^\$/M, \qquad [2.27]$$

which with [2.4] gives the expected permit price

$$p^\$ = (E^b - X^\$)/M \quad (\text{also} > 0). \qquad [2.28]$$

2.6 Optimal, total expected net benefit for a tax and tradable permits

We now derive our main results in Proposition 1, on what a tax and tradable permits achieve for total expected net benefit, where this is the policy objective maximised under risk-neutrality. The derivation assumes up to three separate optimisation processes: parties always optimise their own private abatement decisions $\{\tilde{Q}^j_i\}$; the authority may or may not optimise the set of target indexes $\{\beta^j_i\}$; and it may or may not optimise the expected target total X^j. The first step is to compute in Lemma 1 the risk-neutral, total expected net benefit when parties optimise $\{\tilde{Q}^j_i\}$, but the authority optimises neither $\{\beta^j_i\}$ nor X^j. We present tax results first, to facilitate conventional prices *versus* quantities comparisons.

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3. Also from [2.24] and [2.25], the *realised* permit price $\tilde{p}^T = (E^b - X^T + \tilde{N}_E + \tilde{N}_C)/M$. This can be negative in some realisations, implying a subsidy on emissions or a penalty for holding emissions permits. However, this will generally not be of empirical relevance; in our Section 3 application, it occurs in at most 0.25% of random realisations.

Lemma 1: The risk-neutral, total expected net benefits of abatement are:

for a tax with thresholds:

$$A^{\$} = \bar{A}^{\$} \qquad\qquad + \tfrac{1}{2}\Sigma(1/M_i - W)D_{Ci} \;-\; \Sigma(1/M)D_{CBi} \qquad [2.29]$$

for tradable emission permits (emissions trading):

$$A^T = \bar{A}^T - \tfrac{1}{2}\Sigma(1/M + W)D_{Ei}(x^T{}_i) \;+\; \tfrac{1}{2}\Sigma(1/M_i - 1/M)D_{Ci} \qquad [2.30]$$

where the net benefit under certainty is

$$\bar{A}^j(p^j) := p^jVM - \tfrac{1}{2}M(1+WM)(p^j)^2;\;\; p^j = (E^b - X^j)/M;\;\; j = \$ \text{ or } T. \qquad [2.31]$$

Proofs: See Appendices 1-2.

To compare how the two mechanisms deal with uncertainty, we wish to equalise the certainty-equivalent terms $\bar{A}^{\$}$ and $\bar{A}^T$. The obvious way to do this would be to choose total targets to maximise $A^{\$}$ or A^T, and then show this must yield $X^{\$} = X^T$, and hence $\bar{A}^{\$} = \bar{A}^T$. However, in general this does not work here, because from [2.12] and [2.30], the target distribution $\{x^T{}_i\}$ − not just the total X^T − affects D_{Ei} and hence A^T. So we make the further restriction that for tradable permits, target indexation $\{\beta_i\}$ is either *optimised*, using the following Lemma 2, or *abandoned* (all β_i set to 0).

Lemma 2: For tradable permits, the optimal degree of indexation is

$$\beta_i = \beta_i^* := \alpha_i/x_i \quad \forall i \qquad [2.32]$$

Proof: Written out in full from [2.30], [2.31], [2.26] and [2.12],

$$A^T = [\Sigma(E^b{}_i - x_iE^b{}_i)/M]VM - \tfrac{1}{2}M(1+WM)[\Sigma(E^b{}_i - x_iE^b{}_i)/M]^2$$
$$- \tfrac{1}{2}(1/M+W)\Sigma\{[(\alpha_i - \beta_ix_i)^2\sigma_{Yi}{}^2 + \alpha_i{}^2\sigma_{\eta i}{}^2 + (1-\alpha_i)^2\sigma_{\rho i}{}^2]\, E^b{}_i{}^2\}$$
$$+ \tfrac{1}{2}\Sigma(1/M_i - 1/M)D_{Ci}$$

The $\{\beta_i\}$ here appear only in the $(\alpha_i-\beta_i x_i)^2$ terms, and each of these has a negative coefficient $-½(1/M+W)\sigma_{Yi}^2$. Choosing $\beta_i = \alpha_i/x_i$ minimises each $(\alpha_i-\beta_i x_i)^2$ to zero, and hence maximises A^T, whatever $\{x_i\}$ are chosen. ‖

Targets with $\beta_i = \beta_i^*$ for all i will be called *optimal intensity targets*, or sometimes optimally indexed targets. With optimal indexation, denoted *, or no indexation, denoted ^{0}, i's net emissions uncertainty simplifies from [2.12] respectively to

$$[\alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]\, E^{b\,2}_i =: D^*_{Ei}, \quad \text{or} \tag{2.33}$$

$$[\alpha_i^2\sigma_{Yi}^2 + \alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]\, E^{b\,2}_i =: D^0_{Ei}, \tag{2.34}$$

and these are substituted below for the general D_{Ei}'s in [2.30].

Finally, we move from total expected net benefit under general abatement levels in Lemma 1, to total expected net benefit under *optimal* abatement, denoted †, using the restriction of no or optimal indexation of tradable permit allocations.

Proposition 1. Risk-neutral, total expected net benefit of optimal abatement:

for a tax with any or no indexation of targets ($\beta_i \geq 0\ \forall i$):

$$A^{\$\dagger} = \bar{A} + ½\Sigma(1/M_i-W)D_{Ci} - \Sigma(1/M)D_{CBi} \tag{2.35}$$

for tradable permits with optimally indexed targets ($\beta_i = \alpha_i/x_i\ \forall i$):

$$A^{T*\dagger} = \bar{A} - ½\Sigma(1/M+W)D^*_{Ei} + ½\Sigma(1/M_i-1/M)D_{Ci} \tag{2.36}$$

for tradable permits with unindexed (absolute) targets ($\beta_i = 0\ \forall i$):

$$A^{T0\dagger} = \bar{A} - ½\Sigma(1/M+W)D^0_{Ei} + ½\Sigma(1/M_i-1/M)D_{Ci} \tag{2.37}$$

where the optimal net benefit under certainty is

$$\bar{A} := ½V^2M/(1+WM). \tag{2.38}$$

($\bar{A}$ here and $\bar{p}$, $\bar{Q}$, etcetera below should strictly have †'s added, since they apply only when abatement is optimal, but we omit the † to reduce clutter.)

Interpretation: From [2.34] and [2.33], the first column of uncertainty terms here, with D_{Ei}'s, shows the net cost of uncertainties in BAU emissions, using the slopes $\{1/M_i\}$ of marginal abatement costs from [2.22] and the (downward) slope W of total marginal benefit from [2.21]. From [2.19], the second column, with D_{Ci}'s, shows the net benefit of uncertainties in marginal abatement costs. From [2.20], the third column, with D_{CBi}'s, shows the net cost, in the tax case only, of correlated uncertainties in abatement benefits and costs. Pure benefit uncertainties (terms in σ_{Bi}^{2}) are absent.

Proof: Net uncertainties, $\{D^{*}_{Ei}\}$ in [2.33] or $\{D^{0}_{Ei}\}$ in [2.33], no longer depend on target levels $\{x_i^j\}$. The only effect of $\{x_i^j\}$ on net benefit A^j is then the effect of the total X^j on the emission price p^j in [2.26] or [2.28], and hence on $\bar{A}^j(p^j)$. Maximising $\bar{A}^j(p^j)$ with respect to p^j in [2.31] gives $VM - M(1{+}WM)p^j = 0$, so for either mechanism, j = \$ or T, the optimal price is:

$$p^{\$} = p^{T} = \bar{p} := V/(1{+}WM) \quad \$/\mathrm{t}; \qquad [2.39]$$

and [2.39] with [2.31] gives [2.38], also written as $\bar{A} = ½\bar{p}VM$. ‖

Other certainty-equivalent total expectations will be useful, and from [2.24], [2.25], [2.13] and [2.15] are respectively:

$$\left.\begin{array}{lll} \textit{abatement} & = \bar{Q} := \bar{p}M = VM/(1{+}WM) & \mathrm{t/yr;} \\ \textit{target total} & = \bar{X} := E^{b} - \bar{p}M & \mathrm{t/yr;} \\ \textit{benefit} & = \bar{B} := ½\bar{p}M(\bar{p}{+}V) & \$/\mathrm{yr;} \\ \textit{cost} & = \bar{C} := ½\bar{p}^{2}M & \$/\mathrm{yr;} \end{array}\right) \qquad [2.40]$$

The expected price $\bar{p}$ in [2.39] and total abatement $\bar{Q}$ in [2.40] obey $\bar{p} = V - W\bar{Q} = B'(\bar{Q})$ from [2.21], the Samuelson (1954) rule that the optimal price of a public good under certainty equals its total marginal benefit.

2.7 Discussion of Proposition 1

First, note that since parties are heterogeneous, total expected net benefit depends on parameter distributions $\{M_i\}$, $\{D_{Ci}\}$ and $\{D_{Ei}\}$ as well as on parameter totals W, V and M, but not on n, the number of parties. We group our more detailed comments under two headings.

2.7.1 The value of abatement cost uncertainty, and the difficulty of guessing multi-party from single-party results

In [2.35], define the net effect of uncertainties in marginal abatement cost under a tax as

$$Z^{\$} := \tfrac{1}{2}\Sigma(1/M_i - W)D_{Ci}. \qquad [2.41]$$

From [2.19], slopes $1/M_i = \tilde{C}^{j}_{i}{}''$ from [2.22] and $-W = B^{j}{}''$ from [2.21], and other slight changes in notation, this is exactly Weitzman's result (21) for Δ_n, the advantage of prices versus quantities in the heterogeneous, multi-party (hereafter just 'multi-party') case. However, Proposition 1 changes the interpretation of $Z^{\$}$ in two ways.

First, $Z^{\$}$ comes from just the tax result. So if there was no benefit-cost correlation ($D_{CBi} = 0$), $Z^{\$}$ is the net *benefit* of uncertainty, compared to no uncertainty (a comparison not made directly by Weitzman). It is indeed positive benefit if W is low enough, a result worth noting as:

Corollary 1: With a tax, marginal abatement cost uncertainty ***increases*** *expected total net benefit compared to certainty, provided the total marginal benefit curve has less slope than a weighted mean of slopes of the marginal cost curves.*

The proviso here is true for short-run GHG analyses, where the slope W of the total marginal benefit curve is very low (Pizer 2002).

Second, even with no benefit-cost correlation or emissions uncertainty ($D_{CBi} = D_{Ei} = 0$), $Z^\$$ is not the advantage of prices over tradable quantities. From [2.36] and [2.37], this is $Z^\$-Z^T$, where

$$Z^T := \tfrac{1}{2}\Sigma(1/M_i - 1/M)D_{Ci}. \quad [2.42]$$

We will consider the differences between the mechanisms below. Note, though, Z^T *must* be positive, since $M = \Sigma M_k > M_i$,[4] hence:

Corollary 2: With tradable permits, marginal abatement cost uncertainty ***increases*** *expected total net benefit compared to certainty.*

Intuitively, Z^T is different from $Z^\$$ because the benefit to party i from a low realised marginal abatement cost both is proportional to the square of the shift in marginal abatement cost,[5] and with permit trading is shared with all other parties through a common but uncertain permit price; whereas such sharing does not take place under a tax. But the form of Z^T is hard to intuit from any single-party model, where Z^T disappears. Likewise, if there is benefit-cost correlation ($D_{CBi} > 0$), it is not straightforward to guess that the multi-party generalisation of $(1/M_i)D_{CiBi}$ (our notation for the extra term in

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4. Typically $1/M_i >> 1/M$, and for our global GHG case in Section 3 with 18 parties, even $\min\{1/M_i\}$ (for China) $> 4/M$.

5. If the benefit of q is q^2, then an equal chance of $q+\varepsilon$ or $q-\varepsilon$ is better than a certain q, since $\tfrac{1}{2}[(q+\varepsilon)^2+(q-\varepsilon)^2] - q^2 = \varepsilon^2 > 0$.

Weitzman's p485 footnote, as highlighted by Stavins 1996), is $(1/\Sigma M_i)(\Sigma D_{CiBi})$, not $\Sigma(1/M_i)D_{CiBi}$ or $(\Sigma 1/M_i)(\Sigma D_{CiBi})$. Hence:

Corollary 3: An empirical, multi-party comparison of a tax versus tradable quantities under uncertainty may be quite inaccurate if based on a single-party model.

Our GHG case-study in Section 3 gives an example of how bad the inaccuracy might be (see also Section 2.8 below).

2.7.2 Differences between market mechanisms

The differences between [2.35], [2.36] and [2.37] are:

Advantage of a tax over tradable permits with optimally indexed targets:

$$A^{\$\dagger} - A^{T*\dagger} = ½(1/M-W)D_C + ½(1/M+W)D^*_E - (1/M)D_{CB} \qquad [2.43]$$

Advantage of a tax over tradable permits with absolute targets:

$$A^{\$\dagger} - A^{T0\dagger} = ½(1/M-W)D_C + ½(1/M+W)D^0_E - (1/M)D_{CB} \qquad [2.44]$$

Advantage of optimally indexed over absolute tradable permits:

$$A^{T*\dagger} - A^{T0\dagger} = ½(1/M)(D^0_E - D^*_E) = \Sigma\alpha_i^2\sigma_{Yi}^2 E^{b\,2}_i > 0. \qquad [2.45]$$

A tax eliminates the cost of emissions uncertainties (so indexing tax targets gives no extra benefit), while tradable permits don't, so a higher W now has an ambiguous effect in [2.43] and [2.44]. It lowers, has opposing effects on, or raises the advantages of price over the two quantity mechanisms, depending on whether $D_C > D^0_E > D^*_E$, $D^0_E > D_C > D^*_E$, or $D^0_E > D^*_E > D_C$ (the last being our GHG case, with D_C much the smallest term). We highlight this as:

Corollary 4: When emissions as well as marginal abatement costs are uncertain, the advantage of a tax over tradable permits is more complex than the prices vs. quantities difference in Weitzman (1974), and a more steeply sloping marginal abatement curve (higher W) no longer always favours control by quantities (permits).

By contrast, [2.45], derived from [2.33] and [2.34], shows that optimally indexed tradable permits are always better than absolute tradable permits, because of the activity-linked emissions uncertainty that the former neutralise.

2.8 The importance of permit trading

Here we further highlight the importance of using a multi-party model with permit trading, by showing how much worse the cost of uncertainty will be if permits are non-tradable, and using the result as a rough approximation for uncertainty under quantity control as modelled for a single party.[6] We do this with our version of the formal *non-tradable* permits in Weitzman (1974, Section V), denoted $^{@}$ here. Under these, the authority requires each party's realised, abated emissions $\tilde{E}^{@}_i$ to equal its non-tradable permit target $\tilde{X}^{@}_i$ (indexed as in [2.9], and with $X^{@} < E^b$):

$$\tilde{E}^{@}_i = \tilde{X}^{@}_i \quad \text{t/yr,} \qquad [2.46]$$

where in order to maximise total expected net benefit:

each target $X^{@}_i$ is chosen so that i's *expected* marginal abatement cost equals a common shadow price, say $p^{@}$ \$/t (> 0). [2.47]

6. We are not interested here in the well-known, though distribution-dependent and thus much debated, value of permit tradability under certainty (Weyant 1999).

These rules differ from conventional regulatory (i.e. non-tradable) permits or standards, since [2.46] is an equality constraint, not the usual maximum constraint, $\tilde{E}^{@}_{i} \leq \tilde{X}^{@}_{i}$, and also choice [2.47] is based solely on efficiency, not equity, grounds. Nevertheless, this formal mechanism will show the extra uncertainty costs that [2.46] imposes, by not allowing permit trading under the common price $\tilde{p}^{T}$ to cushion each party's deviations from expected emissions. Expected net benefit for non-tradable permits can be shown (in calculations available from the authors *[here in Referees' Appendix 3]*) to be:

Proposition 2. Expected, optimal, total net benefit for non-tradable permits:

with optimal intensity targets:

$$A^{@*\dagger} = \bar{A} - \tfrac{1}{2}\Sigma(1/M_i+W)D^{*}_{Ei} \qquad [2.48]$$

with absolute targets:

$$A^{@0\dagger} = \bar{A} - \tfrac{1}{2}\Sigma(1/M_i+W)D^{0}_{Ei} \qquad [2.49]$$

Comparing these non-tradable results with the tradable results in [2.36] and [2.37], the advantage of permit tradability is respectively

$$\left.\begin{array}{ll} & A^{T*\dagger}-A^{@*\dagger} = \tfrac{1}{2}\Sigma(1/M_i-1/M)(D^{*}_{Ei}+D_{Ci}) \\ \text{or} & A^{T0\dagger}-A^{@0\dagger} = \tfrac{1}{2}\Sigma(1/M_i-1/M)(D^{0}_{Ei}+D_{Ci}) \end{array}\right) \qquad [2.50]$$

here, showing how trading cushions uncertainties in both emissions (D_{Ei}) and abatement costs (D_{Ci}). Either advantage is large if $M >> M_i$, which is generally true in any practical trading system split into many major parties. We thus have a stronger version of Corollary 3:

Corollary 5: Insofar as a single-party model approximates the net benefit of tradable permits to formal non-tradable permits, it will greatly overstate the relative advantage of a tax over tradable permits.

The degree to which such overstatement happen will obviously vary greatly, depending on the application and specification of models, but at least for our GHG case in Section 3, such overstatement can indeed be large.

2.9 Deducting the benefit from unilateral action

When we come to present empirical results in the next section for GHG control by countries, we choose not to report expected net benefit, A, but a more policy-relevant measure. This recognises that if there is no treaty, country i is motivated to abate *unilaterally* (denoted ^{U}) by some amount $\tilde{Q}^U{}_i$, so as to maximise its unilateral net benefit, which from [2.23] is

$$\tilde{A}^U{}_i := (V_i+\tilde{N}_{Bi}/M)\tilde{Q}^U - ½W_i(\tilde{Q}^U)^2 - ½(1/M_i)(\tilde{Q}^U{}_i)^2 - (1/M_i)\tilde{Q}^U{}_i\tilde{N}_{Ci}, \quad [2.51]$$

while making the non-cooperative assumption that $\partial\tilde{Q}^U/\partial\tilde{Q}^U{}_i = 1$ (compared to the zero conjecture made under a treaty that $\partial\tilde{Q}^U/\partial\tilde{Q}^U{}_i = 0$). What a climate treaty rather than no treaty achieves is then its realised *gain in net benefit*, denoted $\tilde{G}^j{}_i$, from abating under mechanism j instead of unilaterally:

$$\tilde{G}^j{}_i := \tilde{A}^j{}_i - \tilde{A}^U{}_i. \quad [2.52]$$

So below we report expected total gain, $G^j = A^j - A^U$, using the following expected net benefit results under unilateralism (derived in Appendix 3):

$$A^U = \bar{A}^U + F^U \text{ where} \quad [2.53]$$

$$\bar{A}^U := \Sigma\ \{V_i\bar{Q}^U - ½W_i(1+W_iM_i)(\bar{Q}^U)^2 - ½V_i^2M_i + V_iW_iM_i\bar{Q}^U\}, \quad [2.54]$$

$$F^U = ½\Sigma\ \{(1/M_i)D_{Ci} - W_i(1+W_iM_i)[D_C/(1+\Sigma W_kM_k)^2\}, \text{ and} \quad [2.55]$$

$$\bar{Q}^U := \Sigma V_iM_i/(1+\Sigma W_kM_k). \quad [2.56]$$

3. EMPIRICAL RESULTS FOR GLOBAL GREENHOUSE GAS CONTROL

Here we apply the MATES model to a hypothetical, global, cooperative treaty for greenhouse gas (GHG) abatement in 2020. We use parameter values from the empirical model in Jotzo and Pezzey, which divides the globe into 18 regions or countries, hereafter known just as countries, which range in GDP from Argentina and Australia to the USA and Europe.[7] Section 3.1 gives global results that assume all countries are risk-neutral, and thus use the analytical expectation formulae in Section 2, but leave the distribution of targets across countries indeterminate. Section 3.2 gives global results under the more realistic assumption that all countries are risk-averse. These have to be calculated numerically, using an equity criterion that determines target distribution among countries, and allow mechanism choice to affect optimal levels of abatement as well as welfare.

3.1 Risk-neutral results

The expected, optimal global gains in net benefit under risk-neutrality,

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7. Key global parameters for the model are as follows, with \$ = US\$ in 2000 and t = tonne of CO_2-equivalent. Global population, denoted L, = 7.7×10^9; global GDP, Y = 86.3×10^{12} \$/yr; global BAU emissions of GHGs, E^b = 54.4 Gt/yr (= $E_{2002} \times 1.30$); weighted average share of emissions linked with GDP (= energy sector share), $(\Sigma E^b{}_i\alpha_i)/E^b = 0.64$; global linear valuation of abatement, V = 21.9 \$/t; slope of global marginal abatement benefit, W = 0.22 \$.yr/G(t^2); inverse of sum of slopes of marginal abatement costs, $1/M$ = 2.32 \$.yr/G(t^2); global scaled uncertainties in marginal slopes of marginal abatement costs, D_C = 0.37 (Gt/yr)^2; global scaled uncertainties in BAU emissions with absolute targets, $D_E{}^0$ = 11.35 (Gt/yr)^2, and with optimal intensity targets, $D_E{}^*$ = 7.82 (Gt/yr)^2; abatement benefit/cost correlation, D_{CB} = 0 (assumed because there is no evidence for correlation in the GHG case).

$G^{j\dagger} = A^{j\dagger} - A^U$, are reported in **Table 1** for a tax, tradable permits and non-tradable permits, using [2.35], [2.36], [2.37], [2.48] or [2.49] for $A^{j\dagger}$, the net benefit under the treaty, and [2.53]-[2.56] for A^U, the net benefit under unilateral action.

Table 1 Results for optimal GHG abatement under risk-neutrality (maximisation of expected global net benefit)

(% figures are differences from the case of absolute tradable permits)

<table>
<tr><td rowspan="3">Mechanism</td><td colspan="2">Expected global gain in net benefit vs. unilateralism (G$/yr)</td><td colspan="2">Expected global abatement (Gt/yr)</td></tr>
<tr><td colspan="4">Type of targets</td></tr>
<tr><td>Absolute</td><td>Optimal Intensity</td><td>Absolute</td><td>Optimal Intensity</td></tr>
<tr><td>Certainty case: any mechanism</td><td colspan="2">$\bar{G}^{\dagger}(\bar{Q})$
81.6 (+15.6%)</td><td colspan="2" rowspan="4">$\bar{Q}$
8.62
(15.9% of E^b)</td></tr>
<tr><td>Tax with Thresholds</td><td colspan="2">$G^{\$\dagger}(\bar{Q})$
85.4 (+21.0%)</td></tr>
<tr><td>Tradable Permits</td><td>$G^{T0\dagger}(\bar{Q})$
70.6 (0.0%)</td><td>$G^{T*\dagger}(\bar{Q})$
75.1 (+6.4%)</td></tr>
<tr><td>Non-Tradable Permits</td><td>$G^{@0\dagger}(\bar{Q})$
−64.3 (−191%)</td><td>$G^{@*\dagger}(\bar{Q})$
−18.0 (−125%)</td></tr>
<tr><td colspan="5">All cases have expected $\bar{p}$ = 20 $/t, abatement benefit $\bar{B}$ = 180.6 G$/yr, cost $\bar{C}$ = 86.2 G$/yr (0.1% of Y), and net benefit $\bar{A}$ = 94.4 G$/yr (from [2.39], [2.40] and [2.38]). Unilateral case has abatement $\bar{Q}^U$ = 0.48 Gt/yr (0.9% of E^b), and expected net benefits of $\bar{A}^U(\bar{Q}^U)$ = 8.95 G$/yr under certainty, and $A^U(\bar{Q}^U)$ = 12.7 G$/yr under uncertainty.</td></tr>
</table>

From [2.40], there is no difference in global abatement among the mechanisms. Abatement $\bar{Q}$ of 8.6 Gt/yr is 16% of BAU emissions, and abated emissions in 2020 still grow from the 2000 baseline. This is a modest level of global abatement, reflected in a certainty element of abatement cost, $\bar{C}$, that is only 0.1% of projected global GDP in 2020, reassuring us that this is an acceptable application of a mainly partial equilibrium model like MATES to this case study.

Subject to the crucial proviso that the emission tax includes thresholds as in Pezzey (2003), so it can be made politically equivalent to free tradable permits, the key differences among mechanisms are in gain $G^{j\dagger}$ in net benefit, caused by differences in the way mechanisms cope with uncertainty. Consider first net benefit $A^{\$\dagger}$ from the tax. From $W << 1/M_i$ and with no correlation (D_{CBi}'s = 0) in [2.35], we know $A^{\$\dagger}$ exceeds the certainty net benefit $\bar{A} = ½\bar{p}VM$, so the tax is clearly the best policy under uncertainty, with an extra 21% of gain compared to absolute tradable permits. If a tax is off the policy agenda, we are left with the 6.4% improvement in gain available by switching from absolute, tradable permits to optimal intensity, tradable permits.

The importance of permit tradability in coping with uncertainty is evident from the very poor performance of formal non-tradable permits. Even though this mechanism only approximates real-world non-tradable permits, it shows how very much worse − indeed, harmful in this case − a climate treaty could be if countries cannot trade permits, so that divergences from emission expectations in any country cannot be cushioned by such trade. It thus illustrates Corollary 5, that a single-party prices vs. quantities model can seriously overestimate the advantages of a tax over tradable permits, by effectively misclassifying the tradability advantage [2.50] as part of the prices vs. (tradable) quantities advantage [2.43] or [2.44].

3.2 Risk-averse results

Here we introduce further policy realism by incorporating a country's risk aversion, which we feel gives an even better guide to how countries might assess the practical appeal of a potential climate treaty. We define:

− party i's *payoff*, the welfare it perceives from cooperatively enacting a treaty using abatement mechanism j instead of acting unilaterally, as the gain in net benefit between the two states, plus a strictly concave function of the difference in net benefit:

$$\tilde{U}^j_i(\tilde{G}^j_i) := \tilde{G}^j_i + z_i(1-e^{-r_i\tilde{G}^j_i}) \ \ \$/\text{yr}; \ \ z_i \ \$/\text{yr} > 0, \ \ r \ \text{yr}/\$ > 0. \qquad [3.1]$$

Because of the non-linearity this introduces, we now explicitly assume that the error distributions, so far unspecified, are normal:

$$\varepsilon_{Yi} \sim N(0,\sigma_{Yi}), \ \ \varepsilon_{\eta i} \sim N(0,\sigma_{\eta i}), \ \ \varepsilon_{\rho i} \sim N(0,\sigma_{\rho i}), \ \ \varepsilon_{Ci} \sim N(0,\sigma_{Ci}). \qquad [3.2]$$

Defining $\varepsilon := (\varepsilon_{Y1},...,\varepsilon_{Yn},\varepsilon_{\eta 1},...,\varepsilon_{\eta n},\varepsilon_{\rho 1},...,\varepsilon_{\rho n},\varepsilon_{C1},...,\varepsilon_{Cn})$, the expectation of any random variable $\tilde{J}$ is then

$$J := \int_{\mathbb{R}^{4n}} \tilde{J}(\varepsilon)\ \mathbf{[}e^{-\frac{1}{2}\Sigma[(\varepsilon_{Yi}/\sigma_{Yi})^2+(\varepsilon_{\eta i}/\sigma_{\eta i})^2+(\varepsilon_{\rho i}/\sigma_{\rho i})^2+(\varepsilon_{Ci}/\sigma_{Ci})^2]} \\ / \ (2\pi)^{2n}\Pi^n_{i=1}(\sigma_{Yi}\sigma_{\eta i}\sigma_{\rho i}\sigma_{Ci})\mathbf{]}\ d\varepsilon \qquad [3.3]$$

As shown in an earlier version of this paper (Pezzey and Jotzo 2006), it is then possible to find analytic, matrix-based formulae for a country's expected payoff U^j_i for a tax or tradable permits, and hence for global expected payoff U^j. However, unconstrained maximisation of U^j turns out to result in a supposedly optimal target distribution, $\{x^{j\dagger}_i\}$, which is politically nonsensical: many countries are allocated absurdly high or low targets, depending on how they are affected by uncertainty and risk aversion (an outcome which however would remain unobservable in a single-party model). For policy realism we therefore introduce an *equity criterion* for a sensible target distribution:

> For any mechanism j, targets $\{x^j_i\}$ are distributed to equalise each country's expected payoff per person: $U^j_i/L_i = U^j_k/L_k$ for all i and k, where L_i and L_k are their populations. [3.4]

Expected global payoff U^j is then maximised subject to this criterion, as discussed further in Jotzo and Pezzey.

Using any equity criterion like [3.4], analytical formulae for optimal global payoff $U^{j\dagger}$ can no longer be found. **Table 2** gives numerically-derived results for GHG abatement in 2020, using the extra calibration that the risk aversion parameters in [3.1] are $z_i = L_i/Y_i$ $\$^2/\text{yr}^2$.person, and $r_i = 0.085$ yr/\$ for all i. The results confirm the advantage of a tax over tradable permits, and under risk aversion this gives a greater relative boost than before: for optimal intensity targets, an added 22 percentage points, as against about 15 percentage points under risk neutrality. The results also show that under risk aversion, optimal total abatement is *endogenous*: choosing an abatement mechanism which reduces the equity-constrained, risk-adjusted cost of uncertainty entails cost savings being partly 'spent' on more abatement, which in turn yields the optimal expected payoff. Abatement is increased by 27% by moving from absolute tradable permits, to either a tax with absolute targets, or optimal intensity tradable permits. Note also that optimal indexation of tax targets improves the performance of price-based policies, but only slightly.

The endogeneity of abatement under risk-aversion also explains the absence of non-tradable permits from Table 2. Optimising expected global payoff with such permits would require each party's expected shadow price

Table 2 Results for optimal GHG abatement under risk-aversion (equity-constrained maximisation of expected global payoff)

% figures are differences from absolute tradable permits; z_i = risk aversion weighting parameter = L_i/Y_i $\$^2$/yr^2.person; r_i = risk aversion parameter = 0.085 yr/$

Mechanism	Expected global payoff vs. unilateralism (G$/yr)		Expected global abatement (Gt/yr)	
	Type of targets			
	Absolute	Optimal Intensity	Absolute	Optimal Intensity
Certainty case: any mechanism	$\bar{U}^{\dagger}(\bar{Q})$ 76.6 (61%)		$\bar{Q}$ 8.62 (+33%)	
Tax with Thresholds	$U^{\$0\dagger}(Q^{\$0\dagger})$ 74.7 (+57%)	$U^{\$*\dagger}(Q^{\$*\dagger})$ 75.7 (+59%)	$Q^{\$0\dagger}$ 8.20 (+27%)	$Q^{\$*\dagger}$ 8.46 (+31%)
Tradable Permits	$U^{T0\dagger}(Q^{T0\dagger})$ 47.6	$U^{T*\dagger}(Q^{T*\dagger})$ 65.2 (+37%)	$Q^{T0\dagger}$ 6.46	$Q^{T*\dagger}$ 8.2 (+27%)

to be uniquely endogenous to that party, which then invalidates the assumption [2.47] of a common shadow price that is needed to reach results like [2.48] and [2.49].

However, the most important aspect of our results is found by comparing them with other models of global GHG abatement that are based on single rather than many, heterogeneous parties. Pizer (2002) reported a relative advantage of prices over quantities (that is, a ratio of the welfare gain or net benefits from using a tax instead of tradable permits, both compared to no policy) of about 5. The comparable figure in Newell and Pizer (2003) was between 2.2 and 4.8, depending on the duration of the policy. By contrast, we find a much smaller ratio between about 1.2 and 1.6 (depending on indexation) under risk aversion, and between 1.1 and 1.2 under risk neutrality. This again illustrates Corollary 5, that single-party models inherently overestimate the advantage of prices over quantities.

4. TAX INTERACTION EFFECTS

4.1 Tax Interaction effects on net benefit

Here we allow for 'double dividend', or more accurately 'tax interaction' effects: the effects of the interaction between $\tilde{p}^j$, the price incentive created by mechanism j, and the existing, distortionary tax system in party (now country) i, that creates a marginal excess burden there of μ_i (> 0) per dollar of tax revenue raised. Adding tax interaction effects to our model immediately extends the theoretical analysis in Quirion (2004)[8] in two significant ways: it allows for many, heterogeneous parties rather than just one, again capturing the essence of why market mechanisms of control are used in the first place; and it allows targets to be indexed to activity levels.

Another new feature in this section is that only a proportion − rather than all, as in Sections 2 and 3, or all or nothing, as in Quirion and most other writers on tax interaction effects − of each emitter's target is given away free:

ϕ^j_i = the proportion ($0 \le \phi^j_i \le 1$) of the value of i's expected target that i gets free as permits or a threshold, for mechanism j.

Quirion's formula for party i's tax-interaction-adjusted net benefit from abatement mechanism j, originally found in Goulder et al. (1999), then becomes

$$\tilde{A}^j_i := \tilde{B}^j_i - (1+\mu_i)\tilde{C}^j_i - \mu_i \tilde{p}^j \phi^j_i \tilde{X}^j_i. \qquad [4.1]$$

For a tradable permit system, ϕ^T_i would be the proportion of i's target $\tilde{X}^T_i$ that is given away free, or 'grandfathered'; but the total target $\tilde{X}^T$ is

8. Quirion uses μ for the marginal cost of public funds, so his notation in a multi-party setting would be μ_i-1 in place of our μ_i.

unaffected, still equalling total permits created, and hence total abated emissions $\tilde{E}^T$. For a tax, i's threshold would be $\phi^{\$}_i \tilde{X}^{\$}_i$, with the target $\tilde{X}^{\$}_i$ defined as before, so that the total expected target $X^{\$}$ still equals abated emissions $E^{\$}$. With either mechanism, when the parties are countries, they, not the (treaty) authority, would probably decide their own ϕ^j_i; whereas when the parties are firms, the usual assumption would be that the authority determines all the ϕ^j_i's.

Using [4.1] in [2.52] for the gain in net benefit and [3.1] for payoff then allows empirical applications using numerical modelling of many, risk-averse parties, as in the GHG example in Section 3.2. However, we first take the theoretical, risk-neutral analysis further to get more insight on tax interaction effects, and show an important restriction they impose. For this we must make two simplifying assumptions about the tax interaction parameters:

(i) all $\mu_i = \mu$, as would be the case if the parties were similar firms in the same country, and therefore subject to the same existing tax regime;

(ii) all $\phi^j_i = \phi$, to show the overall effect of the amount of free permits or thresholds as a proportion of targets.

Our main result then is transformed as follows.

[contd...]

Proposition 3. Risk-neutral, total expected net benefits of optimal abatement, allowing for tax interaction effects (denoted $_\mu$*), are:*

provided total abatement is desirable enough that $V > \mu\phi E^b/M$*, then* [4.2]

for a tax with any or no indexation of targets:

$$A^{\$\dagger}_\mu = \bar{A}_\mu \qquad\qquad + \tfrac{1}{2}\Sigma[(1+\mu)(1/M_i) - W]D_{Ci} \tag{4.3}$$

for tradable permits with optimally indexed targets:

$$A^{T*\dagger}_\mu = \bar{A}_\mu - \tfrac{1}{2}[(1+\mu)/M + W]D^*_E + \tfrac{1}{2}\Sigma(1+\mu)(1/M_i - 1/M)D_{Ci} \tag{4.4}$$

for tradable permits with unindexed (absolute) targets:

$$A^{T0\dagger}_\mu = \bar{A}_\mu - \tfrac{1}{2}[(1+\mu)/M + W]D^0_E + \tfrac{1}{2}\Sigma(1+\mu)(1/M_i - 1/M)D_{Ci} \tag{4.5}$$

where the optimal net benefit under certainty is

$$\bar{A}_\mu := \tfrac{1}{2}(V - \mu\phi E^b/M)^2 M/[1+(1-2\phi)\mu + WM] \quad \$/yr. \tag{4.6}$$

Hence

Advantage of a tax over tradable permits with optimally indexed targets:

$$A^{\$\dagger}_\mu - A^{T*\dagger}_\mu = \tfrac{1}{2}[(1+\mu)/M - W]D_C + \tfrac{1}{2}[(1+\mu)/M + W]D^*_E \tag{4.7}$$

Advantage of a tax over tradable permits with absolute targets:

$$A^{\$\dagger}_\mu - A^{T0\dagger}_\mu = \tfrac{1}{2}[(1+\mu)/M - W]D_C + \tfrac{1}{2}[(1+\mu)/M + W]D^0_E \tag{4.8}$$

Advantage of optimally indexed over absolute tradable permits:

$$A^{T*\dagger}_\mu - A^{T0\dagger}_\mu = \tfrac{1}{2}[(1+\mu)/M](D^0_E - D^*_E). \tag{4.9}$$

Proof: Available from the authors' website *[here in Referees' Appendix 4]*.

Other certainty-equivalent results here are

$$\textit{price} \quad = \bar{p}_\mu := (V - \mu\phi E^b/M)/[1+(1-2\phi)\mu + WM] \quad \$/t; \text{ and} \tag{4.10}$$

$$\textit{abatement} \quad = \bar{Q}_\mu := \bar{p}_\mu M = (V - \mu\phi E^b/M)M/[1+(1-2\phi)\mu + WM] \text{ t/yr.} \tag{4.11}$$

4.2 Discussion of Proposition 3

First, in the above generalisations of Proposition 1, all $1/M_i$ and $1/M$ terms (but not W terms) have been multiplied by $1+\mu$, V has been replaced by $V-\mu\phi E^b/M$, and $1+WM$ has been replaced by $1+(1-2\phi)\mu+WM$. From comparing the difference results [4.7]-[4.9] with [2.43]-[2.45], it is the first of these three changes that matters for mechanism choice, giving:

Corollary 6: The marginal excess burden μ makes a tax relatively more attractive than a tradable permit system (though if abatement cost uncertainty D_C and marginal benefit slope W are large enough, tradable permits may still be better absolutely); and it also increases the advantage of optimally indexed over absolute tradable permits.

Second, an expression for expected total net benefit under certainty as a function of a general emission price p can be shown to be:

$$\bar{A}_\mu(p) \quad = M(V-\mu\phi E^b/M)p - \tfrac{1}{2}M[1+(1-2\phi)\mu+WM]p^2 \qquad [4.12]$$

This function is maximised algebraically when $\bar{A}_\mu'(p) = 0$, provided $\tfrac{1}{2}M[1+(1-2\phi)\mu+WM] > 0$. This inequality is very likely to hold, since $WM > 0$ and $\phi \leq 1$ by definition, and few empirical estimates of μ exceed 1, so then $(1-2\phi)\mu > -1$. However, we need $\bar{A}_\mu'(0) > 0$, that is, $V > \mu\phi E^b/M$ or *condition [4.2], for initial abatement to raise rather than lower net benefit,* a phenomenon often stressed in tax interaction literature like Goulder et al. (1997, 1999).

In our empirical application to global GHG control in Section 3, using the 'benchmark' figure of $\mu = 0.3$ from Goulder et al (1999) gives $\mu\phi E^b/M = 37.8\phi$ \$/t, compared to $V = 21.9$ \$/t. So [4.2] is not met if all permits and thresholds are distributed freely ($\phi = 1$) as in Sections 2-3. Proposition 3 shows that ϕ affects only the total expected net benefit under certainty, not (from [4.7]-[4.9]) the differences between mechanisms under uncertainty; but we need to choose ϕ low enough to satisfy [4.2].[9] For our GHG application with $\mu = 0.3$, this means

$$\phi \leq 21.9/37.8 \approx 0.6 \qquad [4.13]$$

Proportions of free permits much lower than this limit were derived for the GHG case by Bovenberg and Goulder (2001) to meet the criterion of leaving fossil fuel producers' profits unchanged. Our model does not include profits, but as a pragmatic guide the limit [4.13] seems thoroughly justified, even though in most countries, climate policy debate has not yet accepted the idea that mostly free permits represents an inequitable distribution.

4.3 Empirical, risk-neutral, tax-interaction-inclusive results for greenhouse gases

Table 3 shows selected results for the expected global net benefit of GHG abatement in 2020, using the same calibration as before. The first column of results corresponds to the results for gain in net benefit in Table 1, where there are no tax interaction effects. The other columns show the effect of a marginal excess burden $\mu = 0.3$, with the proportion ϕ of free permits or thresholds rising from 0 to 0.25 and 0.5. Both higher μ and higher ϕ reduce the net benefit and the abatement optimally achieved by each mechanism. From [4.7]-[4.8], a higher marginal excess burden μ improves the advantage of a tax over tradable permits, but a higher proportion ϕ of free permits or thresholds leaves it unchanged.

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9. An alternative way to justify an assumption that total abatement is beneficial would be to challenge the empirical validity of the basic tax-interaction net-benefit formula [4.1] (Goodstein 2003).

Table 3 Results for GHG abatement mechanisms under risk-neutrality, with tax interaction effects (maximisation of expected global net benefit)

Marginal excess burden	$\mu = 0$	$\mu = 0.3$	$\mu = 0.3$	$\mu = 0.3$
Free permits or thresholds as proportion of total target	$\phi = 0$	$\phi = 0$	$\phi = 0.25$	$\phi = 0.5$
Net benefit of tax with thresholds, $A^{\$\dagger}_{\mu}$ (G$/yr)	98.1	79.0	31.7	6.6
Net benefit of absolute tradable permits, $A^{T0\dagger}_{\mu}$ (G$/yr)	83.3	60.1	12.8	−12.2
Extra benefit from tax instead of absolute tradable permits, $A^{\$\dagger}_{\mu} - A^{T\dagger 0}_{\mu}$ (G$/yr)	14.8	18.9		
Abatement under either mechanism, Q_{μ} (Gt/yr)	8.6	6.8	4.3	1.2

5. CONCLUSIONS

Over the last two decades there has been a slow but seemingly inexorable increase in policymakers' acceptance of the need to use created market mechanisms to control public goods and bads, including the world's most costly future pollution problem, from greenhouse gas emissions. Yet surprisingly, there has been no accurate theory of the comparative performance under uncertainty of the best-known such mechanisms: prices (a tax) and *tradable* quantities (tradable permits). We have provided the first extension of Weitzman's 1974 seminal work that analyses tradable permits in their realistic setting of many parties with heterogeneous marginal control costs. Our model, where public harm or benefit depends only on the total quantity of 'emissions', applies to many situations other than emissions control, such as controlling consumers' total demands for a physically scarce commodity like water. As a further generalisation of Weitzman's model, we have included three different uncertainties in each party's business-as-usual emissions as well as one uncertainty in marginal abatement costs, in a predominantly partial equilibrium model. We have also allowed a party's permit allocation − or threshold above which the tax is paid, needed so that a tax might have the same political acceptability as tradable permits − to be flexibly indexed to its activity level; and we included optimal indexation.

Our main theoretical results are inevitably more complex than Weitzman's, and show the need for heterogeneous-party modelling in several ways. With tradable permits, marginal abatement cost uncertainties are an inherent *benefit* compared to certainty (an effect which vanishes in a single-party model); while under a tax, they are an inherent benefit if the slope of the marginal abatement benefit curve is low enough. The effect of this slope on the advantage of a tax over tradable permits is ambiguous, depending on the relative sizes of abatement cost uncertainty and emissions uncertainty, so a steep marginal benefit curve no longer necessarily favours ‘control by quantities’. Optimal intensity targets (indexed to activity levels) always outperform absolute targets, though. Not surprisingly, tradable permits perform much better than Weitzman’s pure form of non-tradable permits, because permit trading cushions the effect of both emissions and abatement cost uncertainties. Hence single-party models may seriously overestimate the advantage of a tax over tradable permits, if their ‘quantity’ mechanism in fact behaves closer to such non-tradable permits.

Empirical results were derived from applying our model to global greenhouse gas abatement in 2020 with an 18-region calibration. We modelled both risk-neutral behaviour, where welfare is defined as expected global net benefit compared to unilateral behaviour, and our theoretical results can be used; and risk-averse behaviour, where welfare is a strictly concave function of net benefit, and is maximised numerically subject to an equity criterion across countries. Under risk neutrality, a tax gives about 15 to 20% more welfare than tradable permits, depending on whether or not targets are indexed to GDP. All three mechanisms give vastly more welfare than formal non-tradable permits, showing the potential inaccuracy of single-party comparative results. Under risk aversion, using an abatement mechanism which neutralises more uncertainty will increase abatement as well as welfare. We then find the advantage of prices over quantities is larger, with a tax giving up to 60% more welfare than tradable permits, depending on indexation. So empirically, an emission tax remains the best mechanism for greenhouse gas control, but the advantage from price control is not nearly as large as suggested by single-country models.

A final extension to include tax interaction ('double dividend') effects showed the need for strict limits on either tax thresholds or free permits. Both in order for abatement actually to raise welfare, and for equity reasons, the proportion of such handouts should be well under a half in the greenhouse case. Tax interaction effects increase the advantage of taxes over tradable permits, and of optimal intensity targets over absolute targets.

Our model can be applied to many further strands of empirical work. A first is for other pollutants which are completely mixed, or have been treated as such, like sulphur dioxide in the USA. A second is where access to a total resource pool must be restricted, as with water allocations, or some cases of fish or timber stocks. A third strand, with work under way, is analysis of variant market mechanisms, such as non-binding tradable permits, and hybrids between price and quantity control.

APPENDIX 1 NET BENEFIT FROM TAX WITH THRESHOLDS

From [2.23] and [2.5], party *i*'s realised net benefit under a tax with thresholds, compared to no abatement anywhere, is

$$\tilde{A}^\$_i \quad := (V_i+\tilde{N}_{Bi}/M)\tilde{Q}^\$ - ½W_i(\tilde{Q}^\$)^2 - ½(1/M_i)(\tilde{Q}^\$_i)^2 - (1/M_i)\tilde{Q}^\$_i\tilde{N}_{Ci}$$

$$+ p^\$[\tilde{X}^\$_i-\tilde{E}^b_i+\tilde{Q}^\$_i(p^\$)] - (X^\$_i/X^\$)p^\$[\tilde{X}^\$-\tilde{E}^b+\tilde{Q}^\$(p^\$)] \qquad [A1.1]$$

Now from [2.27] and [2.10]

$$\tilde{X}^\$_i-\tilde{E}^b_i+\tilde{Q}^\$_i(p^\$) = p^\$M_i+X^\$_i-E^b_i-\tilde{N}_{Ei}-\tilde{N}_{Ci}, \qquad [A1.2]$$

and summing this and using [2.28] gives

$$\tilde{X}^\$-\tilde{E}^b+\tilde{Q}^\$(p^\$) = p^\$M+X^\$-E^b-\tilde{N}_E-\tilde{N}_C = -(\tilde{N}_E+\tilde{N}_C). \qquad [A1.3]$$

Combining [A1.1]−[A1.3] then gives after some algebra available from the authors *[here in Referees' Appendix 1]*:

$$\tilde{A}^\$_i = \bar{A}^\$_i + \tilde{F}^\$_i - p^\$(\tilde{N}_{Ei}+\tilde{N}_{Ci}-\tilde{N}_{Bi}) + H^\$_{Ei}\tilde{N}_E + H^\$_{Ci}\tilde{N}_C, \text{ where} \qquad [A1.4]$$

$$\bar{A}^\$_i \quad := V_ip^\$M - ½W_i(p^\$)^2M^2 + ½(p^\$)^2M_i - p^\$(E^b_i-X^\$_i) \qquad [A1.5]$$

$$\tilde{F}^\$_i \quad := ½(1/M_i)\tilde{N}_{Ci}^{\,2} - ½W_i\tilde{N}_C^{\,2} - \tilde{N}_C\tilde{N}_{Bi}/M \qquad [A1.6]$$

$$H^\$_{Ei} \quad := (X^\$_i/X^\$)p^\$ \qquad [A1.7]$$

$$H^\$_{Ci} \quad := (X^\$_i/X^\$)p^\$ - V_i + W_ip^\$M \qquad [A1.8]$$

Summing and taking expectations gives total expected net benefit:

$$A^\$ \quad = \bar{A}^\$ + ½\Sigma(1/M_i-W)D_{Ci} - \Sigma(1/M)D_{CBi} \qquad \text{as in [2.29],}$$

where (also using [2.28])

$$\bar{A}^\$ \quad := p^\$VM - ½(p^\$)^2M(1+WM) \qquad \text{as in [2.31].}$$

APPENDIX 2 NET BENEFIT FROM TRADABLE PERMITS

From [2.23], party *i*'s realised net benefit under tradable permits is

$$\tilde{A}^T_i \quad := (V_i+\tilde{N}_{Bi}/M)\tilde{Q}^T - ½W_i(\tilde{Q}^T)^2 + \tilde{R}^T_i - ½(1/M_i)\tilde{Q}^{T\,2}_i - (1/M_i)\tilde{Q}^T_i\tilde{N}_{Ci}, \qquad \text{[A2.1]}$$

while from [2.3], [2.24] and [2.11], permit sales revenue is

$$\tilde{R}^T_i = \tilde{p}^T(\tilde{p}^T M_i - \tilde{N}_{Ci} - E^b_i + X^T_i - \tilde{N}_{Ei}), \qquad \text{[A2.2]}$$

and from [2.25] and [2.26],

$$\tilde{Q}^T = p^T M + \tilde{N}_E. \qquad \text{[A2.3]}$$

Combining [A2.1] with [A2.2], [A2.3] and [2.24] then gives

$$\begin{aligned}\tilde{A}^T_i \quad &= (V_i+\tilde{N}_{Bi}/M)(p^T M+\tilde{N}_E) - ½W_i(p^T M+\tilde{N}_E)^2 \\ &\quad + \tilde{p}^T(\tilde{p}^T M_i-\tilde{N}_{Ci}-E^b_i+X^T_i-\tilde{N}_{Ei}) \\ &\quad - ½(1/M_i)(\tilde{p}^T M_i-\tilde{N}_{Ci})^2 - (1/M_i)(\tilde{p}^T M_i-\tilde{N}_{Ci})\tilde{N}_{Ci}, \end{aligned} \qquad \text{[A2.4]}$$

which can be shown by lengthy algebra available from the authors *[here in Referees' Appendix 2]* to be

$$\tilde{A}^T_i \quad = \bar{A}^T_i + \tilde{F}^T_i - p^T(\tilde{N}_{Ei}+\tilde{N}_{Ci}-\tilde{N}_{Bi}) + H^T_{Ei}\tilde{N}_E + H^T_{Ci}\tilde{N}_C \qquad \text{[A2.5]}$$

where

$$\bar{A}^T_i \quad := V_i p^T M + ½(p^T)^2(M_i-W_i M^2) - p^T(1-x_i)E^b_i \ \ \text{\$/yr}, \qquad \text{[A2.6]}$$

$$\begin{aligned}\tilde{F}^T_i \quad &:= ½(M_i/M^2-W_i)\tilde{N}_E^{\,2} - (1/M)\tilde{N}_{Ei}^{\,2} + ½(M_i/M^2)\tilde{N}_C^{\,2} \\ &\quad + [½(1/M_i)-1/M]\tilde{N}_{Ci}^{\,2} + (M_i/M^2)\tilde{N}_E\tilde{N}_C \\ &\quad - (1/M)[\tilde{N}_{Ei}(\tilde{N}_{E-i}+\tilde{N}_C)+\tilde{N}_{Ci}(\tilde{N}_{C-i}+\tilde{N}_E)+\tilde{N}_{Bi}\tilde{N}_E] \end{aligned} \qquad \text{[A2.7]}$$

$$H^T_{Ei} \ := (p^T M_i - E^b_i + X^T_i)/M + V_i - W_i p^T M, \ \text{ and} \qquad \text{[A2.8]}$$

$$H^T_{Ci} \ := (p^T M_i - E^b_i + X^T_i)/M. \qquad \text{[A2.9]}$$

Taking expectations and summing over all parties then gives total expected net benefit:

$$A^T = \bar{A}^T + F^T, \text{ where}$$

$$\bar{A}^T = Vp^T M + ½(p^T)^2(M-WM^2) - p^T(E^b-X^T), \text{ which with [2.26] gives}$$

$$\bar{A}^T = p^T VM - ½M(1+WM)(p^T)^2 \qquad \text{as in [2.31], and}$$

$$F^T := ½(1/M-W)\Sigma D_{Ei} - (1/M)\Sigma D_{Ei} + ½(1/M)\Sigma D_{Ci} + ½\Sigma(1/M_i)D_{Ci} - (1/M)\Sigma D_{Ci}$$

$$= -½\Sigma(1/M+W)D_{Ei} + ½\Sigma(1/M_i-1/M)D_{Ci}. \qquad \text{as in [2.30].}$$

(The result for F^T relies on the independence of all $\{\tilde{N}_{Ei}\}$ from $\{\tilde{N}_{Ci}\}$ and $\{\tilde{N}_{Bi}\}$.)

APPENDIX 3 NET BENEFIT FROM UNILATERALISM

To compute the unilateral case, party i chooses $\tilde{Q}^U{}_i$ to maximise

$$\tilde{A}^U{}_i := V_i\tilde{Q}^U - ½W_i(\tilde{Q}^U)^2 - ½(1/M_i)(\tilde{Q}^U{}_i)^2 - (1/M_i)\tilde{Q}^U{}_i\tilde{N}_{Ci} \qquad \text{[A3.1]}$$

([2.51] with the benefit uncertainty term $(\tilde{N}_{Bi}/M)\tilde{Q}^U$ omitted for simplicity), on the assumption that $\partial\tilde{Q}^U/\partial\tilde{Q}^U{}_i = 1$.[10] Hence

$$\partial\tilde{A}^U{}_i/\partial\tilde{Q}^U{}_i = V_i - W_i\tilde{Q}^U - (1/M_i)\tilde{Q}^U{}_i - (1/M_i)\tilde{N}_{Ci} = 0$$

$$\Rightarrow \tilde{Q}^U{}_i = V_iM_i - \tilde{N}_{Ci} - W_iM_i\tilde{Q}^U \qquad \text{[A3.2]}$$

---

10. For formal consistency, $(\tilde{N}_{Bi}/M)\tilde{Q}^U$ should be kept, but it greatly complicates the algebra here while making a negligible empirical difference in Section 3. And for simplicity we also choose [A3.1] for the risk-averse as well as the risk-neutral case. Under risk aversion i should maximise not its expected net benefit, but its expected payoff using [3.1]. However, the latter maximisation has no analytic solution and differs only very slightly from the former.

$\Rightarrow\ \tilde{Q}^U = \Sigma(V_iM_i - \tilde{N}_{Ci}) \,/\, (1+\Sigma W_kM_k)$

$\Rightarrow\ Q^U = \mathrm{E}[\tilde{Q}^U] = \Sigma V_iM_i/(1+\Sigma W_kM_k) =: \bar{Q}^U$ as in [2.56].

$\Rightarrow\ \mathrm{E}[(\tilde{Q}^U)^2] = [\Sigma V_iM_i/(1+\Sigma W_kM_k)]^2 + \Sigma\ \mathrm{E}[\tilde{N}_{Ci}{}^2]/(1+\Sigma W_kM_k)^2$

$= [(\Sigma V_iM_i)^2 + D_C]/(1+\Sigma W_kM_k)^2 = (\bar{Q}^U)^2 + D_C/(1+\Sigma W_kM_k)^2;$

$\mathrm{E}[\tilde{N}_{Ci}\tilde{Q}^U] = -\ \mathrm{E}\ (\tilde{N}_{Ci})^2/(1+\Sigma W_kM_k) = -\ D_{Ci}/(1+\Sigma W_kM_k).$

So from [A3.1] and [A3.2],

$$\tilde{A}^U{}_i \ := \ V_i\tilde{Q}^U - ½W_i(\tilde{Q}^U)^2 - ½(1/M_i)(V_iM_i - \tilde{N}_{Ci} - W_iM_i\tilde{Q}^U)^2$$
$$- (1/M_i)(V_iM_i - \tilde{N}_{Ci} - W_iM_i\tilde{Q}^U)\tilde{N}_{Ci}; \qquad \text{[A3.3]}$$

$$\Rightarrow\ A^U{}_i = V_i\bar{Q}^U - ½W_i[(\bar{Q}^U)^2 + D_C/(1+\Sigma W_kM_k)^2] - ½V_i^2M_i - ½(1/M_i)D_{Ci}$$
$$- ½W_i^2M_i[(\bar{Q}^U)^2 + D_C/(1+\Sigma W_kM_k)^2] + V_iW_iM_i\bar{Q}^U$$
$$+ W_iD_{Ci}/(1+\Sigma W_kM_k) + (1/M_i)D_{Ci} - W_iD_{Ci}/(1+\Sigma W_kM_k)$$

$= \bar{A}^U{}_i + F^U{}_i$ where as in [2.53]

$\bar{A}^U{}_i := V_i\bar{Q}^U - ½W_i(1+W_iM_i)(\bar{Q}^U)^2 - ½V_i^2M_i + V_iW_iM_i\bar{Q}^U,$ as in [2.54]

$F^U{}_i = -\ ½W_i(1+W_iM_i)[D_C/(1+\Sigma W_kM_k)^2] + ½(1/M_i)D_{Ci}$ as in [2.55].

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REFEREES' APPENDICES FOR "CONTROLLING TOTAL EMISSIONS UNDER UNCERTAINTY" BY PEZZEY & JOTZO, 4/07

Referees' Appendix 1 Detailed algebra for tax with thresholds

From [A1.1] with [2.27], [A1.2] and [A1.3] (see Referees' Appendix 5 for reminder of notation),

$$\begin{aligned}
\tilde{A}^\$_i &:= (V_i+\tilde{N}_{Bi}/M)(p^\$M-\tilde{N}_C) - ½W_i(p^\$M-\tilde{N}_C)^2 + p^\$(p^\$M_i+X^\$_i-E^b_i-\tilde{N}_{Ei}-\tilde{N}_{Ci}) \\
&\qquad + (X^\$_i/X^\$)p^\$(\tilde{N}_E+\tilde{N}_C) - ½(1/M_i)(p^\$M_i-\tilde{N}_{Ci})^2 - (1/M_i)(p^\$M_i-\tilde{N}_{Ci})\tilde{N}_{Ci} \\
&= V_ip^\$M + p^\$\tilde{N}_{Bi} - ½W_i(p^\$)^2M^2 - (V_i+\tilde{N}_{Bi}/M-W_ip^\$M)\tilde{N}_C - ½W_i\tilde{N}_C^{\,2} \\
&\quad + (p^\$)^2M_i - p^\$(E^b_i-X^\$_i) - p^\$\tilde{N}_{Ei} - p^\$\tilde{N}_{Ci} + (X^\$_i/X^\$)p^\$(\tilde{N}_E+\tilde{N}_C) \\
&\quad - ½(p^\$)^2M_i + p^\$\tilde{N}_{Ci} - (1/2M_i)\tilde{N}_{Ci}^{\,2} - p^\$\tilde{N}_{Ci} + (1/M_i)\tilde{N}_{Ci}^{\,2} \\
&= V_ip^\$M - ½W_i(p^\$)^2M^2 + ½(p^\$)^2M_i - p^\$(E^b_i-X^\$_i) - (V_i-W_ip^\$M)\tilde{N}_C \\
&\quad - p^\$(\tilde{N}_{Ei}+\tilde{N}_{Ci}-\tilde{N}_{Bi}) + (X^\$_i/X^\$)p^\$(\tilde{N}_E+\tilde{N}_C) - ½W_i\tilde{N}_C^{\,2} - ½(1/M_i)\tilde{N}_{Ci}^{\,2} \\
&\quad + (1/M_i)\tilde{N}_{Ci}^{\,2} - \tilde{N}_C\tilde{N}_{Bi}/M \\
&= V_ip^\$M - ½W_i(p^\$)^2M^2 + ½(p^\$)^2M_i - p^\$(E^b_i-X^\$_i) \\
&\quad + ½(1/M_i)\tilde{N}_{Ci}^{\,2} - ½W_i\tilde{N}_C^{\,2} - \tilde{N}_C\tilde{N}_{Bi}/M \\
&\quad - p^\$(\tilde{N}_{Ei}+\tilde{N}_{Ci}-\tilde{N}_{Bi}) + (X^\$_i/X^\$)p^\$\tilde{N}_E - [V_i-W_ip^\$M-(X^\$_i/X^\$)p^\$]\tilde{N}_C.
\end{aligned}$$

hence $\tilde{A}^\$_i = \bar{A}^\$_i + \tilde{F}^\$_i - p^\$(\tilde{N}_{Ei}+\tilde{N}_{Ci}-\tilde{N}_{Bi}) + H^\$_{Ei}\tilde{N}_E + H^\$_{Ci}\tilde{N}_C$ as in [A1.4]

where $\bar{A}^\$_i := V_ip^\$M - ½W_i(p^\$)^2M^2 + ½(p^\$)^2M_i - p^\$(E^b_i-X^\$_i)$ as in [A1.5]

$\tilde{F}^\$_i := ½(1/M_i)\tilde{N}_{Ci}^{\,2} - ½W_i\tilde{N}_C^{\,2} - \tilde{N}_C\tilde{N}_{Bi}/M,$ as in [A1.6]

$H^\$_{Ei} := (X^\$_i/X^\$)p^\$,$ as in [A1.7]

and $H^\$_{Ci} := (X^\$_i/X^\$)p^\$ - V_i + W_ip^\$M$ as in [A1.8].

Referees' Appendix 2 Detailed algebra for tradable permits

From [A2.4]:

$$\begin{aligned}
\tilde{A}^T_i &= (V_i+\tilde{N}_{Bi}/M)(p^TM+\tilde{N}_E) - ½W_i(p^TM+\tilde{N}_E)^2 + \tilde{p}^T(\tilde{p}^TM_i-\tilde{N}_{Ci}-E^b_i+X^T_i-\tilde{N}_{Ei}) \\
&\qquad - ½(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 - (1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})\tilde{N}_{Ci}
\end{aligned}$$

$$= (V_i+\tilde{N}_{Bi}/M)p^TM - ½W_i(p^T)^2M^2 + (V_i+\tilde{N}_{Bi}/M-W_ip^TM)\tilde{N}_E - ½W_i\tilde{N}_E{}^2$$
$$+ (1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 + (1/M_i)\tilde{N}_{Ci}(\tilde{p}^TM_i-\tilde{N}_{Ci}) - \tilde{p}^T(E^b{}_i-X^T{}_i+\tilde{N}_{Ei})$$
$$- ½(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 - (1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})\tilde{N}_{Ci}$$
$$= (V_i+\tilde{N}_{Bi}/M)p^TM - ½(p^T)^2W_iM^2 - p^T(E^b{}_i-X^T{}_i)$$
$$+ [V_i+\tilde{N}_{Bi}/M-W_ip^TM-(E^b{}_i-X^T{}_i)/M]\tilde{N}_E - ½W_i\tilde{N}_E{}^2$$
$$+ ½(p^TM+\tilde{N}_E+\tilde{N}_C)^2M_i/M^2 - \tilde{N}_{Ci}(p^TM+\tilde{N}_E+\tilde{N}_C)/M + ½(1/M_i)\tilde{N}_{Ci}{}^2$$
$$- p^T\tilde{N}_{Ei} - (\tilde{N}_C/M)(E^b{}_i-X^T{}_i) - [(\tilde{N}_E+\tilde{N}_C)/M]\tilde{N}_{Ei}$$
$$= (V_i+\tilde{N}_{Bi}/M)p^TM + ½(p^T)^2(M_i-W_iM^2) - p^T(E^b{}_i-X^T{}_i)$$
$$+ [p^TM_i/M+V_i+\tilde{N}_{Bi}/M-W_ip^TM-(E^b{}_i-X^T{}_i)/M]\tilde{N}_E + ½(M_i/M^2-W_i)\tilde{N}_E{}^2$$
$$+ (p^TM_i/M)\tilde{N}_C + (M_i/M^2)(\tilde{N}_E\tilde{N}_C+½\tilde{N}_C{}^2) - p^T\tilde{N}_{Ci} - (1/M)(\tilde{N}_E+\tilde{N}_C)\tilde{N}_{Ci}$$
$$+ ½(1/M_i)\tilde{N}_{Ci}{}^2 - p^T\tilde{N}_{Ei} - [(E^b{}_i-X^T{}_i)/M]\tilde{N}_C - (1/M)\tilde{N}_{Ei}(\tilde{N}_E+\tilde{N}_C)$$
$$= V_ip^TM + ½(p^T)^2(M_i-W_iM^2) - p^T(E^b{}_i-X^T{}_i)$$
$$+ [p^TM_i/M+V_i-W_ip^TM-(E^b{}_i-X^T{}_i)/M]\tilde{N}_E + (p^TM_i/M)\tilde{N}_C - [(E^b{}_i-X^T{}_i)/M]\tilde{N}_C$$
$$- p^T\tilde{N}_{Ei} - p^T\tilde{N}_{Ci} + p^T\tilde{N}_{Bi} + ½(M_i/M^2-W_i)\tilde{N}_E{}^2 - (1/M)\tilde{N}_{Ei}\tilde{N}_E$$
$$+ (M_i/M^2)(½\tilde{N}_C{}^2) + ½(1/M_i)\tilde{N}_{Ci}{}^2 - (1/M)\tilde{N}_C\tilde{N}_{Ci} + (M_i/M^2)\tilde{N}_E\tilde{N}_C$$
$$- (1/M)\tilde{N}_E\tilde{N}_{Ci} - (1/M)\tilde{N}_{Ei}\tilde{N}_C + \tilde{N}_E\tilde{N}_{Bi}/M.$$

Hence $\tilde{A}^T{}_i = \bar{A}^T{}_i + \tilde{F}_i - p^T(\tilde{N}_{Ei}+\tilde{N}_{Ci}-\tilde{N}_{Bi}) + H^T{}_{Ei}\tilde{N}_E + H^T{}_{Ci}\tilde{N}_C$ as in [A2.5]

where $\bar{A}^T{}_i := V_ip^TM + ½(p^T)^2(M_i-W_iM^2) - p^T(1-x_i)E^b{}_i$ \$/yr, as in [A2.6]

$\tilde{F}^T{}_i := ½(M_i/M^2-W_i)\tilde{N}_E{}^2 - (1/M)\tilde{N}_{Ei}{}^2 + ½(M_i/M^2)\tilde{N}_C{}^2 + [½(1/M_i)-1/M]\tilde{N}_{Ci}{}^2$
$+ (M_i/M^2)\tilde{N}_E\tilde{N}_C - (1/M)[\tilde{N}_{Ei}(\tilde{N}_{E-i}+\tilde{N}_C)+\tilde{N}_{Ci}(\tilde{N}_{C-i}+\tilde{N}_E)+\tilde{N}_{Bi}\tilde{N}_E]$ as in [A2.7]

$H^T{}_{Ei} := (p^TM_i-E^b{}_i+X^T{}_i)/M + V_i - W_ip^TM$, and as in [A2.8]

$H^T{}_{Ci} := (p^TM_i-E^b{}_i+X^T{}_i)/M$, as in [A2.9].

Referees' Appendix 3 Net benefit from non-tradable permits

From [2.47] and the expectation of [2.22]:

$(1/M_i)Q^@{}_i = p^@ \quad \forall i$

$\Rightarrow \quad Q^@ = p^@M = E^b - X^@$ [R3.1]

$\Rightarrow \quad p^@ = (E^b-X^@)/M.$ [R3.2]

Then from [2.46], [2.10] and [R3.1], i's realised abatement is:

$$\tilde{Q}^@_i = \tilde{E}^b_i - \tilde{X}^@_i = E^b_i - X^@_i + \tilde{N}_{Ei} = Q^@ + \tilde{N}_{Ei} = p^@ M_i + \tilde{N}_{Ei}. \qquad \text{[R3.3]}$$

From [2.23] with j = @ and $\tilde{R}^@_i = 0$, party i's realised net benefit is

$$\tilde{A}^@_i = (V_i + \tilde{N}_{Bi}/M)\tilde{Q}^@ - ½W_i(\tilde{Q}^@)^2 - ½(1/M_i)(\tilde{Q}^@_i)^2 - (1/M_i)\tilde{Q}^@_i\tilde{N}_{Ci}, \qquad \text{[R3.4]}$$

which from [R3.3] and its sum, $\tilde{Q}^@ = p^@M + \tilde{N}_E$, is

$$= (V_i + \tilde{N}_{Bi}/M)(p^@M + \tilde{N}_E) - ½W_i(p^@M + \tilde{N}_E)^2 - ½(1/M_i)(p^@M_i + \tilde{N}_{Ei})^2$$
$$- (1/M_i)(p^@M_i + \tilde{N}_{Ei})\tilde{N}_{Ci}$$
$$= (V_i + \tilde{N}_{Bi}/M)p^@M - ½W_i(p^@)^2M^2 - ½M_i(p^@)^2$$
$$+ (V_i + \tilde{N}_{Bi}/M - W_ip^@M)\tilde{N}_E - p^@(\tilde{N}_{Ei} + \tilde{N}_{Ci})$$
$$- ½W_i\tilde{N}_E^2 - ½(1/M_i)\tilde{N}_{Ei}^2 - ½(1/M_i)\tilde{N}_{Ei}\tilde{N}_{Ci}, \text{ so}$$
$$= V_ip^@M - ½W_i(p^@)^2M^2 - ½M_i(p^@)^2$$
$$- ½W_i\tilde{N}_E^2 - ½(1/M_i)\tilde{N}_{Ei}^2 - ½(1/M_i)\tilde{N}_{Ei}\tilde{N}_{Ci} + (1/M)\tilde{N}_E\tilde{N}_{Bi}$$
$$- p^@(\tilde{N}_{Ei} + \tilde{N}_{Ci} - \tilde{N}_{Bi}) + (V_i - W_ip^@M)\tilde{N}_E, \text{ so}$$

$$\tilde{A}^@_i = \bar{A}^@_i + \tilde{F}^@_i - p^@(\tilde{N}_{Ei} + \tilde{N}_{Ci} - \tilde{N}_{Bi}) + H^@_{Ei}\tilde{N}_E, \text{ where}$$
$$\bar{A}^@_i := V_ip^@M - ½(M_i + W_iM^2)(p^@)^2,$$
$$\tilde{F}^@_i := -½W_i\tilde{N}_E^2 - ½(1/M_i)\tilde{N}_{Ei}^2 - ½(1/M_i)\tilde{N}_{Ei}\tilde{N}_{Ci} + (1/M)\tilde{N}_E\tilde{N}_{Bi},$$
$$\& \quad H^@_{Ei} := V_i - W_ip^@M.$$

Taking expectations and summing then gives

$$A^@ = \bar{A}^@ - ½\Sigma(1/M_i + W)D_{Ei}, \text{ where} \qquad \text{[R3.5]}$$
$$\bar{A}^@ := p^@VM - ½M(1 + WM)(p^@)^2,$$

This result again uses the assumed independence of all $\{\tilde{N}_{Ei}\}$ from $\{\tilde{N}_{Ci}\}$ and $\{\tilde{N}_{Bi}\}$; and as in Lemma 2, we need to restrict the analysis to optimal target indexation (denoted *) or no target indexation (absolute targets, ^{0}) to be able to compute an optimal (†) level of total abatement. This gives $p^@ = \bar{p}$ in [2.39], $\bar{A}^@ = \bar{A}$ in [2.38], and hence

$$A^{@*\dagger} = \bar{A} - ½\Sigma(1/M_i + W)D^*_{Ei}, \qquad \text{as in [2.48], and}$$
$$A^{@0\dagger} = \bar{A} - ½\Sigma(1/M_i + W)D^0_{Ei} \qquad \text{as in [2.49].}$$

Referees' Appendix 4 Net benefit with tax interaction effects

Combining [4.1], [2.13], [2.15] and [2.18] gives

$$\tilde{A}^j_i = V_i\tilde{Q}^j - ½W_i(\tilde{Q}^j)^2 - (1+\mu)\,[½(1/M_i)(\tilde{Q}^j_i)^2 + (1/M_i)\tilde{Q}^j_i\tilde{N}_{Ci} - \tilde{R}^j_i] - \mu\tilde{p}^j\phi\tilde{X}^j_i \qquad \text{[R4.1]}$$

We follow the same procedure as in Appendices 1 and 2, and omit the $_\mu$ subscripts to save clutter. We also define

$$\tilde{N}_{Xi} := \beta_i x_i \varepsilon_{Yi} E^b_i, \;\text{ so that }\; \tilde{X}_i = X_i + \tilde{N}_{Xi} \qquad \text{[R4.2]}$$

For a *Tax with Thresholds:*

Combining [R4.1] with [2.27], [2.5] and [2.10] gives

$$\begin{aligned}
\tilde{A}^\$_i \;&:= V_i(p^\$M-\tilde{N}_C) - ½W_i(p^\$M-\tilde{N}_C)^2 + (1+m)\,\mathbf{[}p^\$(p^\$M_i+X^\$_i-E^b_i-\tilde{N}_{Ei}-\tilde{N}_{Ci}) \\
&\qquad + (X^\$_i/X^\$)p^\$(\tilde{N}_E+\tilde{N}_C) - ½(1/M_i)(p^\$M_i-\tilde{N}_{Ci})^2 - (1/M_i)(p^\$M_i-\tilde{N}_{Ci})\tilde{N}_{Ci}\mathbf{]} - mp^\$\phi^\$_i\tilde{X} \\
&= V_ip^\$M - ½W_i(p^\$)^2M^2 - (V_i-W_ip^\$M)\tilde{N}_C - ½W_i\tilde{N}_C{}^2 \\
&\quad + (1+m)\mathbf{[}(p^\$)^2M_i - p^\$(E^b_i-X^\$_i) - p^\$\tilde{N}_{Ei} - p^\$\tilde{N}_{Ci} + (X^\$_i/X^\$)p^\$(\tilde{N}_E+\tilde{N}_C) \\
&\quad - ½(p^\$)^2M_i + p^\$\tilde{N}_{Ci} - (1/2M_i)\tilde{N}_{Ci}{}^2 - p^\$\tilde{N}_{Ci} + (1/M_i)\tilde{N}_{Ci}{}^2\mathbf{]} - mp^\$\phi^\$_i(X^\$_i+\tilde{N}_{Xi}) \\
&= V_ip^\$M - ½W_i(p^\$)^2M^2 + ½(1+m)(p^\$)^2M_i - (1+m)p^\$(E^b_i-X^\$_i) - (V_i-W_ip^\$M)\tilde{N}_C \\
&\quad - (1+m)p^\$(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + (1+m)(X^\$_i/X^\$)p^\$(\tilde{N}_E+\tilde{N}_C) - ½W_i\tilde{N}_C{}^2 + ½(1+m)(1/M_i)\tilde{N}_{Ci}{}^2 \\
&\quad - mp^\$\phi^\$_i(X^\$_i+\tilde{N}_{Xi}) \\
&= V_ip^\$M - ½W_i(p^\$)^2M^2 + ½(1+m)(p^\$)^2M_i - (1+m)p^\$(E^b_i-X^\$_i) - mp^\$\phi^\$_iX^\$_i \\
&\quad + ½(1+m)(1/M_i)\tilde{N}_{Ci}{}^2 - ½W_i\tilde{N}_C{}^2 - (1+m)p^\$(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + (1+m)(X^\$_i/X^\$)p^\$\tilde{N}_E \\
&\quad + [-V_i+W_ip^\$M+(1+m)(X^\$_i/X^\$)p^\$]\tilde{N}_C - mp^\$\phi^\$_i\tilde{N}_{Xi}
\end{aligned}$$

So

$$\tilde{A}^\$_i = \bar{A}^\$_i + \tilde{F}^\$_i - p^\$[(1+\mu)(\tilde{N}_{Ei}+\tilde{N}_{Ci})+\mu\phi\tilde{N}_{Xi}] + H^\$_{Ei}\tilde{N}_E + H^\$_{Ci}\tilde{N}_C$$

where

$$\begin{aligned}
\bar{A}^\$_i \;&:= V_ip^\$M - ½W_i(p^\$)^2M^2 + (1+\mu)[½(p^\$)^2M_i-p^\$(E^b_i-X^\$_i)] - \mu p^\$\phi X^\$_i \\
\tilde{F}^\$_i \;&:= ½(1+\mu)(1/M_i)\tilde{N}_{Ci}{}^2 - ½W_i\tilde{N}_C{}^2, \\
H^\$_{Ei} \;&:= (1+\mu)(X^\$_i/X^\$)p^\$, \;\text{ and} \\
H^\$_{Ci} \;&:= (1+\mu)(X^\$_i/X^\$)p^\$ - V_i + W_ip^\$M.
\end{aligned}$$

Summing and taking expectations gives

$$A^{\$\dagger}_{\mu} \quad = \bar{A}^{\$} + \tfrac{1}{2}\Sigma[(1+\mu)(1/M_i)-W]D_{Ci}, \qquad \text{as in [4.3]}$$

where

$$\bar{A}^{\$} \quad := p^{\$}VM - \tfrac{1}{2}(p^{\$})^2M(1+\mu+WM) - \phi\mu p^{\$}X^{\$} = \bar{A}_{\mu} \text{ optimally,} \qquad \text{[R4.3]}$$

(see end of Appendix) with no difference between absolute and optimal intensity targets.

For *Tradable Permits:*

From [2.3], [2.10] and [2.25],

$$\tilde{R}^T_i = \tilde{p}^T(\tilde{p}^TM_i-\tilde{N}_{Ci}-E^b_i+X^T_i-\tilde{N}_{Ei}). \qquad \text{[R4.4]}$$

Combining [R4.1] with [R4.2], [R4.4], [A2.3] and [2.25] then gives

$$\begin{aligned}
\tilde{A}^T_i \; &= V_i(p^TM+\tilde{N}_E) - \tfrac{1}{2}W_i(p^TM+\tilde{N}_E)^2 + (1+\mu)\tilde{p}^T(\tilde{p}^TM_i-\tilde{N}_{Ci}-E^b_i+X^T_i-\tilde{N}_{Ei}) \\
&\qquad - \tfrac{1}{2}(1+\mu)(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 - (1+\mu)(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})\tilde{N}_{Ci} \\
&\qquad - \mu\phi^T_i\tilde{p}^T(X_{Ti}+\tilde{N}_{Xi}) \\
&= V_ip^TM - \tfrac{1}{2}W_i(p^T)^2M^2 + (V_i-W_ip^TM)\tilde{N}_E - \tfrac{1}{2}W_i\tilde{N}_E^{\,2} \\
&\quad + (1+\mu)(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 + (1+\mu)(1/M_i)\tilde{N}_{Ci}(\tilde{p}^TM_i-\tilde{N}_{Ci}) - (1+\mu)\tilde{p}^T(E^b_i-X^T_i+\tilde{N}_{Ei}) \\
&\quad - \tfrac{1}{2}(1+\mu)(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 - (1+\mu)(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})\tilde{N}_{Ci} \\
&\quad - \mu\phi^T_i\tilde{p}^T(X_{Ti}+\tilde{N}_{Xi}) \\
&= V_ip^TM - \tfrac{1}{2}W_i(p^T)^2M^2 + (V_i-W_ip^TM)\tilde{N}_E - \tfrac{1}{2}W_i\tilde{N}_E^{\,2} \\
&\quad + \tfrac{1}{2}(1+\mu)(1/M_i)[(p^TM+\tilde{N}_E+\tilde{N}_C)M_i/M-\tilde{N}_{Ci}]^2 - (1+\mu)[(p^TM+\tilde{N}_E+\tilde{N}_C)/M](E^b_i-X^T_i+\tilde{N}_{Ei}) \\
&\quad - \mu\phi^T_i[(p^TM+\tilde{N}_E+\tilde{N}_C)/M](X_{Ti}+\tilde{N}_{Xi}) \\
&= V_ip^TM - \tfrac{1}{2}(p^T)^2W_iM^2 + (V_i-W_ip^TM)\tilde{N}_E - \tfrac{1}{2}W_i\tilde{N}_E^{\,2} \\
&\quad + \tfrac{1}{2}(1+\mu)(1/M_i)[(p^TM+\tilde{N}_E+\tilde{N}_C)M_i/M-\tilde{N}_{Ci}]^2 \\
&\quad - (1+\mu)[(p^TM+\tilde{N}_E+\tilde{N}_C)/M](E^b_i-X^T_i) - (1+\mu)[(p^TM+\tilde{N}_E+\tilde{N}_C)/M]\tilde{N}_{Ei} \\
&\quad - \mu\phi^T_ip^TX^T_i - \mu\phi^T_ip^T\tilde{N}_{Xi} - \mu\phi^T_i[(\tilde{N}_E+\tilde{N}_C)/M](X_{Ti}+\tilde{N}_{Xi}) \\
&= V_ip^TM - \tfrac{1}{2}(p^T)^2W_iM^2 - (1+\mu)p^T(E^b_i-X^T_i) + [V_i-W_ip^TM-(1+\mu)(E^b_i-X^T_i)/M]\tilde{N}_E \\
&\quad - \tfrac{1}{2}W_i\tilde{N}_E^{\,2} + \tfrac{1}{2}(1+\mu)(p^TM+\tilde{N}_E+\tilde{N}_C)^2M_i/M^2 - (1+\mu)\tilde{N}_{Ci}(p^TM+\tilde{N}_E+\tilde{N}_C)/M \\
&\quad + \tfrac{1}{2}(1+\mu)(1/M_i)\tilde{N}_{Ci}^{\,2} \\
&\quad - (1+\mu)p^T\tilde{N}_{Ei} - (1+\mu)(\tilde{N}_C/M)(E^b_i-X^T_i) - (1+\mu)[(\tilde{N}_E+\tilde{N}_C)/M]\tilde{N}_{Ei} \\
&\quad - \mu\phi^T_ip^TX^T_i - \mu\phi^T_ip^T\tilde{N}_{Xi} - \mu\phi^T_i[(\tilde{N}_E+\tilde{N}_C)/M](X_{Ti}+\tilde{N}_{Xi})
\end{aligned}$$

$$
\begin{aligned}
&= V_i p^T M - \tfrac{1}{2}(p^T)^2 W_i M^2 - (1+\mu)p^T(E^b{}_i - X^T{}_i) + [V_i - W_i p^T M - (1+\mu)(E^b{}_i - X^T{}_i)/M]\tilde{N}_E \\
&\quad + \tfrac{1}{2}(-W_i)\tilde{N}_E{}^2 + \tfrac{1}{2}(1+\mu)((p^T)^2 M^2 + \tilde{N}_E{}^2 + \tilde{N}_C{}^2 + 2p^T M\tilde{N}_E + 2p^T M\tilde{N}_C + 2\tilde{N}_E\tilde{N}_C)M_i/M^2 \\
&\quad - (1+\mu)\tilde{N}_{Ci}(p^T M + \tilde{N}_E + \tilde{N}_C)/M + \tfrac{1}{2}(1+\mu)(1/M_i)\tilde{N}_{Ci}{}^2 \\
&\quad - (1+\mu)p^T\tilde{N}_{Ei} - (1+\mu)(\tilde{N}_C/M)(E^b{}_i - X^T{}_i) - (1+\mu)[(\tilde{N}_E + \tilde{N}_C)/M]\tilde{N}_{Ei} \\
&\quad - \mu\phi^T{}_i p^T X^T{}_i - \mu\phi^T{}_i p^T\tilde{N}_{Xi} - \mu\phi^T{}_i[(\tilde{N}_E + \tilde{N}_C)/M](X_{Ti} + \tilde{N}_{Xi}) \\
&= V_i p^T M + \tfrac{1}{2}(p^T)^2[(1+\mu)M_i - W_i M^2] - (1+\mu)p^T(E^b{}_i - X^T{}_i) \\
&\quad + [(1+\mu)p^T M_i/M + V_i - W_i p^T M - (1+\mu)(E^b{}_i - X^T{}_i)/M]\tilde{N}_E \\
&\quad + \tfrac{1}{2}[(1+\mu)M_i/M^2 - W_i]\tilde{N}_E{}^2 + (1+\mu)(p^T M_i/M)\tilde{N}_C + (1+\mu)(M_i/M^2)(\tilde{N}_E\tilde{N}_C + \tfrac{1}{2}\tilde{N}_C{}^2) \\
&\quad - (1+\mu)p^T\tilde{N}_{Ci} - (1+\mu)(1/M)(\tilde{N}_E + \tilde{N}_C)\tilde{N}_{Ci} + \tfrac{1}{2}(1+\mu)(1/M_i)\tilde{N}_{Ci}{}^2 - (1+\mu)p^T\tilde{N}_{Ei} \\
&\quad - (1+\mu)[(E^b{}_i - X^T{}_i)/M]\tilde{N}_C - (1+\mu)(1/M)\tilde{N}_{Ei}(\tilde{N}_E + \tilde{N}_C) \\
&\quad - \mu\phi^T{}_i p^T X^T{}_i - \mu\phi^T{}_i p^T\tilde{N}_{Xi} - \mu\phi^T{}_i[(\tilde{N}_E + \tilde{N}_C)/M](X_{Ti} + \tilde{N}_{Xi}) \\
&= V_i p^T M + \tfrac{1}{2}(p^T)^2[(1+\mu)M_i - W_i M^2] - (1+\mu)p^T(E^b{}_i - X^T{}_i) - m\phi^T{}_i p^T X^T{}_i \\
&\quad + \tfrac{1}{2}[(1+\mu)M_i/M^2 - W_i]\tilde{N}_E{}^2 + \tfrac{1}{2}(1+\mu)(M_i/M^2)\tilde{N}_C{}^2 + \tfrac{1}{2}(1+\mu)(1/M_i)\tilde{N}_{Ci}{}^2 \\
&\quad + (1+\mu)(M_i/M^2)\tilde{N}_E\tilde{N}_C - (1+\mu)(1/M)\tilde{N}_C\tilde{N}_{Ci} - (1+\mu)(1/M)\tilde{N}_E\tilde{N}_{Ci} \\
&\quad - (1+\mu)(1/M)\tilde{N}_{Ei}\tilde{N}_E - (1+\mu)(1/M)\tilde{N}_{Ei}\tilde{N}_C - (\mu\phi^T{}_i/M)(\tilde{N}_E + \tilde{N}_C)\tilde{N}_{Xi} \\
&\quad - p^T[(1+\mu)(\tilde{N}_{Ei} + \tilde{N}_{Ci}) + \mu\phi^T{}_i\tilde{N}_{Xi}] \\
&\quad + (1/M)[(1+\mu)(p^T M_i - E^b{}_i + X^T{}_i) + \mu\phi^T{}_i X^T{}_i + M(V_i - W_i p^T M)]\tilde{N}_E \\
&\quad + (1/M)[(1+\mu)(p^T M_i - E^b{}_i + X^T{}_i) + \mu\phi^T{}_i X^T{}_i]\tilde{N}_C \\
&= V_i p^T M + \tfrac{1}{2}(p^T)^2[(1+\mu)M_i - W_i M^2] - (1+\mu)p^T(E^b{}_i - X^T{}_i) - \mu\phi^T{}_i p^T X^T{}_i \\
&\quad + \tfrac{1}{2}[(1+\mu)M_i/M^2](\tilde{N}_E + \tilde{N}_C)^2 - \tfrac{1}{2}W_i\tilde{N}_E{}^2 + \tfrac{1}{2}(1+\mu)(1/M_i)\tilde{N}_{Ci}{}^2 \\
&\quad - (1+\mu)(1/M)(\tilde{N}_E + \tilde{N}_C)(\tilde{N}_{Ei} + \tilde{N}_{Ci}) - (\mu\phi^T{}_i/M)(\tilde{N}_E + \tilde{N}_C)\tilde{N}_{Xi} \\
&\quad - p^T[(1+\mu)(\tilde{N}_{Ei} + \tilde{N}_{Ci}) + \mu\phi^T{}_i\tilde{N}_{Xi}] + (V_i - W_i p^T M)\tilde{N}_E \\
&\quad + (1/M)[(1+\mu)(p^T M_i - E^b{}_i + X^T{}_i) + \mu\phi^T{}_i X^T{}_i](\tilde{N}_E + \tilde{N}_C)
\end{aligned}
$$

So

$$\tilde{A}^T{}_i = \bar{A}^T{}_i + \tilde{F}^T{}_i - p^T[(1+\mu)(\tilde{N}_{Ei} + \tilde{N}_{Ci}) + \mu\phi\tilde{N}_{Xi}] + H^T{}_{Ei}\tilde{N}_E + H^T{}_{Ci}\tilde{N}_C,$$

where

$$
\begin{aligned}
\bar{A}^T{}_i &:= V_i p^T M + \tfrac{1}{2}(p^T)^2[(1+\mu)M_i - W_i M^2] - (1+\mu)p^T(E^b{}_i - X^T{}_i) - \mu\phi p^T X^T{}_i \\
\tilde{F}^T{}_i &:= \tfrac{1}{2}[(1+\mu)M_i/M^2](\tilde{N}_E + \tilde{N}_C)^2 - \tfrac{1}{2}W_i\tilde{N}_E{}^2 + \tfrac{1}{2}(1+\mu)(1/M_i)\tilde{N}_{Ci}{}^2 \\
&\qquad - (1+\mu)(1/M)(\tilde{N}_E + \tilde{N}_C)(\tilde{N}_{Ei} + \tilde{N}_{Ci}) - (\mu\phi/M)(\tilde{N}_E + \tilde{N}_C)\tilde{N}_{Xi} \\
H^T{}_{Ei} &:= (1/M)[(1+\mu)(p^T M_i - E^b{}_i + X^T{}_i) + \mu\phi X^T{}_i + M(V_i - W_i p^T M)]
\end{aligned}
$$

$H^T{}_{Ci}$:= $(1/M)[(1+\mu)(p^T M_i - E^b{}_i + X^T{}_i) + \mu\phi X^T{}_i]$
$\tilde{N}_{Xi}$:= $\beta_i x_i \varepsilon_{Yi} E^b{}_i$.

Summing and taking expectations gives

$A^T = \bar{A}^T + F^T$, where

$\bar{A}^T = p^T VM - ½(p^T)^2 M(1+\mu+WM) - \phi\mu p^T X^T$ \$/yr $= \bar{A}_\mu$ optimally (see below) [R4.5]

$F^T := -½\Sigma[(1+\mu)/M+W]D_{Ei} + ½(1+\mu)\Sigma(1/M_i - 1/M)D_{Ci} - \phi(\mu/M)\Sigma D_{EXi}$

$D_{EXi} := \mathrm{E}[\tilde{N}_{Ei}\tilde{N}_{Xi}] = (\alpha_i - \beta_i x_i)\beta_i x_i E^b{}_i{}^2\ \sigma_{Yi}{}^2$

$= 0$ for $\beta_i = \alpha_i/x_i$ or 0

so the variants with optimal intensity or absolute targets are

$A^{T*} = \bar{A}^T - ½\Sigma[(1+\mu)/M+W]D^*{}_{Ei} + ½(1+\mu)\Sigma(1/M_i - 1/M)D_{Ci}$; as in [4.4]

$A^{T0} = \bar{A}^T - ½\Sigma[(1+\mu)/M+W]D^0{}_{Ei} + ½(1+\mu)\Sigma(1/M_i - 1/M)D_{Ci}$. as in [4.5]

-oOo-

Finally, the certainty part of expected total net benefit, $\bar{A}^\$$ in [R4.3] or $\bar{A}^T$ in [R4.5], has the same form, which with $X = E = E^b - Q = E^b - pM$ becomes

$\bar{A}(p) = pVM - ½p^2[WM^2+(1+\mu)M] - \mu\phi E^b p + \mu\phi M p^2$

$= M(V - \mu\phi E^b/M)p - ½M[1+(1-2\phi)\mu+WM]p^2$ as in [4.12]

Given [4.2], the optimal price obeys

$\bar{A}'(p) = M(V-\mu\phi E^b/M) - M[1+(1-2\phi)\mu+WM]p = 0$

$\Rightarrow$ $p = \bar{p}_\mu := (V-\mu\phi E^b/M) / [1+(1-2\phi)\mu+WM]$, as in [4.10]

$Q = \bar{Q}_\mu := (V-\mu\phi E^b/M)M / [1+(1-2\phi)\mu+WM]$, as in [4.11]

& $\bar{A} = \bar{A}^\$ = \bar{A}^T = \bar{A}_\mu := M(V-\mu\phi E^b/M)\bar{p}_\mu - ½M\bar{p}_\mu(V-\mu\phi E^b/M)$

$= ½M(V-\mu\phi E^b/M)^2 / [1+(1-2\phi)\mu+WM]$ as in [4.6]

Referees' Appendix 5 Selected notation used in MATES model

Equation numbers show where the notation is introduced.

$\tilde{}$	denotes random or uncertain variable (with $J := \mathrm{E}[\tilde{J}]$ for any $\tilde{J}$)	[2.1]
$\bar{}$	denotes expectation of equivalent variable under certainty	[2.29]
$^{\$}$	denotes a tax with thresholds	[2.4]
$^{@}$	denotes non-tradable permits	[2.46]
*	denotes optimal indexation of target, $\beta_i = \alpha_i / x_i$	[2.32]
^{0}	denotes no indexation of target, $\beta_i = 0$	[2.34]
†	(qualifying A, G or U) denotes optimal level of total target X	[2.35]
$\tilde{A}^j_i$	$:= \tilde{B}^j_i - \tilde{C}^j_i$ = party i's dollar-valued net benefit (*A*dvantage) from abatement under policy mechanism j [\$/yr]	[2.23]
$\bar{A}^j_i$	party i's net benefit from policy j under certainty	[2.31]
$\tilde{A}^U_i$	party i's dollar-valued net benefit (*A*dvantage) from unilateral abatement by all parties	[2.51]
^{b}	denotes *b*usiness-as-usual (BAU)	[2.1]
$\tilde{B}^j_i$	$:= V_i\tilde{Q}^j - ½W_i(\tilde{Q}^j)^2$ = party i's *B*enefit from total abatement under mechanism j	[2.13]
$\tilde{C}^j_i$	$:= ½(1/M_i){Q^j_i}^2 + Q^j_i\varepsilon_{Ci} - \tilde{R}^j_i$ = party i's *C*ost (net of emissions trading) of its own abatement using mechanism j	[2.15]
D_{Ci}	$:= M_i^2\sigma_{Ci}^2$	[2.19]
D_{CBi}	$:= \Sigma_k M_k M\sigma_{CkBi}^2$	[2.20]
D_{Ei}	$:= {E^b_i}^2[(\alpha_i - \beta_i x_i)^2\sigma_{Yi}^2 + \alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]$	[2.12]
D^0_{Ei}	$:= {E^b_i}^2[\alpha_i^2\sigma_{Yi}^2 + \alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]$	[2.34]
D^*_{Ei}	$:= {E^b_i}^2[\alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]$	[2.33]
$\tilde{E}^j_i$	party i's actual *E*missions under abatement mechanism j	[2.1]
$\tilde{E}^b_i$	party i's realised *b*usiness-As-Usual (unabated) *E*missions	[2.1]
$\tilde{F}^j_i$	part of realised net benefit $\tilde{A}^j_i$ with squared and cross errors	[A1.6]
$\tilde{G}^j_i$	party i's realised *G*ain from mechanism $j := \tilde{A}^j_i - \tilde{A}^U_i$	[2.52]
$_i$ or $_k$	denotes party i or k, one of n parties	[2.1]
^{j}	denotes one of three types of abatement mechanism (^{j} = ^{T}, $^{\$}$, $^{@}$)	[2.1]
L_i	population of party i	[3.4]
M_i	party i's abatement potential (inverse slope of $\tilde{C}^j_i{}'$ curve)	[2.15]
$\tilde{N}_{Bi}$	$:= M\varepsilon_{Bi}$ = weighted marginal abatement benefit uncertainty	[2.18]
$\tilde{N}_{Ci}$	$:= M_i\varepsilon_{Ci}$ = weighted marginal abatement cost uncertainty	[2.18]
$\tilde{N}_{Ei}$	$:= [(\alpha_i - \beta_i x_i)\varepsilon_{Yi} + \alpha_i\varepsilon_{\eta i} + (1-\alpha_i)\varepsilon_{\rho i}]\, E^b_i$.	[2.11]
$\tilde{p}$	common emissions *p*rice or shadow price	[2.3]

$\bar{p}$	$= V/(1+WM)$, optimal emissions *p*rice under certainty	[2.39]
$\tilde{Q}^j_i$	party i's realised abatement under mechanism j	[2.1]
r	*r*isk aversion parameter (curvature of payoff function)	[3.1]
$\tilde{R}^j_i$	party i's net *R*evenue from abatement mechanism j	[2.3]
t	tonne of CO_2-equivalent of GHG	
^{T}	denotes *T*radable permits	[2.2]
$\tilde{U}^j_i$	payoff from i's dollar-valued gain from abatement mechanism j	[3.1]
^{U}	denotes *U*nilateral abatement scenario	[2.51]
V_i	party i's linear benefit (*V*alue) per unit of total abatement $\tilde{Q}^j$	[2.13]
W_i	local (downward) slope of marginal benefit	[2.13]
x_i	$:= X_i/E^b_i$, party i's expected absolute emissions target as proportion of expected BAU emissions	[2.9]
$\tilde{X}_i$	$:= x_iE^b_i(1+\beta_i\varepsilon_{Yi})$ = party i's realised emissions target	[2.9]
$\tilde{Y}_i$	= party i's realised future GDP	[2.6]
z_i	multiplier of risk-averse part of payoff	[3.1]
$Z^\$$	$:= ½\Sigma(1/M_i-W)D_{Ci}$	[2.41]
Z^T	$:= ½\Sigma(1/M_i-1/M)D_{Ci}$	[2.42]
α_i	proportion of economy i in which emissions are GDP-linked	[2.6]
β_i	degree to which i's emissions target is indexed to GDP	[2.9]
β_i^*	optimal β_i ($= \alpha_i/x_i$)	[2.32]
ε_{Ci}	absolute random part of $\tilde{C}_i$	[2.15]
ε_{Yi}	proportional BAU emissions variation caused by GDP fluctuations	[2.6]
$\varepsilon_{\eta i}$	proportional BAU emissions variation caused by intensity fluctuations	[2.6]
$\varepsilon_{\rho i}$	proportional BAU emissions variation caused by other fluctuations	[2.6]
$\tilde{\eta}_i$	party i's BAU emissions intensity = $\tilde{E}^b_i/\tilde{Y}_i$	[2.6]
μ	marginal excess burden of existing taxation (= marginal cost of public funds − 1)	[4.1]
$^\mu$	denotes solution taking tax-interaction effects into account	[4.3]
$^\rho$	denotes part of economy where emissions are independent of GDP and intensity	[2.6]
σ_{Ci}^2	$:= E[\varepsilon_{Ci}^2]$	[2.16]
σ_{CiBk}^2	$:= E[\varepsilon_{Ci}\varepsilon_{Bk}]$	[2.17]
σ_{Yi}^2	$:= E[\varepsilon_{Yi}^2]$	[2.8]
$\sigma_{\eta i}^2$	$:= E[\varepsilon_{\eta i}^2]$	[2.8]
$\sigma_{\rho i}^2$	$:= E[\varepsilon_{\rho i}^2]$	[2.8]