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Abstract. We give theoretical, partial equilibrium comparisons of a tax with thresholds, tradable targets ('emissions trading' or ET), and non-tradable targets, as mechanisms to abate well-mixed ('global') emissions from many parties, under independent uncertainties in both future business-as-usual emissions and marginal abatement costs. All three mechanisms are revenue-neutral, and use flexible thresholds or targets indexed continuously to parties' activity levels. We analyse both risk-neutral or risk-averse behaviour. Key theoretical results are that because of emissions uncertainty, there is no simple Weitzman (1974) rule for choosing between 'prices' (a tax) to 'quantities' (ET); under ET, marginal abatement cost uncertainty is a benefit, compared to certainty; and under risk aversion, any mechanism with more expected welfare also gives more expected abatement. We apply our theory to global greenhouse gas abatement in 2020, using an 18-region numerical simulation model with new uncertainty estimates. Key global, empirical results are that under either risk behaviour, a tax dominates ET, which hugely dominates non-tradable targets; and under risk aversion, an optimally indexed tax gives about 60% more welfare and 30% more abatement than unindexed ET, while optimally indexed ET achieves about two-fifths of these improvements.

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1. INTRODUCTION

The literature comparing control and planning mechanisms under uncertainty has a long history, dating back to Weitzman (1974) and other authors in the early 1970s. By assuming locally quadratic benefit and cost functions with uncorrelated uncertainties for a single producer or polluter in a risk-neutral world, Weitzman showed that 'prices' (or a tax) outperform 'quantities' (a target or regulatory standard) when the marginal cost curve is steeper than the marginal benefit curve at the expected social optimum; and that the degree of outperformance depends solely on the uncertainty in the marginal cost level. Related literature since then on the design of control mechanisms has explored many extensions such as price-quantity hybrids (Roberts and Spence 1976, Pizer 2002), correlation of benefit and cost uncertainty (Stavins 1996) and stock pollutant effects (Hoel and Karp 2002).

In recent years global greenhouse gas (GHG) abatement has inspired many further extensions. The failure of the Kyoto Protocol to involve key developing countries and the USA has been partly attributed to the Protocol's imposition of fixed quantities. This has given rise to the concept of GHG intensity targets, as a variant to alleviate the cost of uncertainty (Ellerman and Sue Wing 2003, and unpublished work by Quirion 2003 and Sue Wing et al 2005). Such literature usually focuses on uncertainties in business-as-usual (BAU) emissions, rather than marginal abatement costs (MACs) as in Weitzman, but an important exception is Quirion (2004), whose main contribution is to introduce the marginal cost of public funds into a Weitzman-style comparison of prices and quantities.

By contrast with the above, we model *any number* of emitting parties, and give parallel analyses, under exogenous uncertainties in BAU emissions and MACs, of three mechanism types: tradable targets, better known as emissions trading (ET); non-tradable targets (NTTs); and an emissions tax. This allows for three different levels of stochasticity in emission prices: a certain global price under the tax; an uncertain global price under ET, which transmits party-level uncertainties globally; and uncertain local shadow prices under Non-Tradable Targets (NTTs). Since we also make several other simultaneous generalisations of the Weitzman framework, so we content ourselves with considering only a 'global' (i.e. uniformly-mixed) pollutant and with price-taking behaviour, so that both non-uniform pollutants and imperfect competition remain for further work. At least as far as existing market-based control schemes go, global pollution includes many pollutants other than GHGs, such as SO_2 in the USA (see for example Joskow and Schmalensee 1998) though none fit the bill as well as GHGs.

In including an emissions tax, we are swimming against the tide of the many policy debates currently focused on ET: for example for GHGs everywhere, now that a carbon tax has been abandoned as a politically viable option, or for almost all well-mixed pollutants in North America. However, it is well known how taxes are in principle preferable to ET for GHGs (Pizer 2002, though he shows that hybrid mechanisms are even better), an insight clear from Weitzman's basic result because of the effective flatness of the marginal benefit curve for long-lived, long-accumulated stock pollutants. To restore the political viability of a tax we make it *revenue-neutral*, with no loss of abatement cost efficiency, by using tax thresholds like property rights (Pezzey 1992), and a novel adjustment mechanism to ensure that revenue-neutrality is still exactly achieved under uncertainty. This mechanism loses welfare compared to a simple revenue-raising tax because of the tax interaction effect (see for example Bovenberg and Goulder 1996, Goulder et al. 1999), but so does ET with free permit allocations, and we wish to compare a tax and ET on a politically level, therefore revenue-neutral, playing field.[1]

We include *risk aversion* by constructing a strictly concave 'payoff function' to introduce diminishing marginal utility to parties of expected dollar amounts. Any such function is hard to quantify, and indeed our modelling involves inference rather than measurement of its key parameters. However, we include it both in the hope of greater political realism; and also because in contrast to a risk-neutral world, it allows any mechanism which increases expected payoff by neutralising more uncertainty to increase the overall stringency of optimal abatement, a phenomenon we call *endogenous targets*. In practice it turns out this requires some *equity criterion* to give a politically sensible target distribution, which also contributes to and informs political realism.

Our contribution to the debate on abatement mechanism choice is best summed up as a simultaneous, fivefold generalisation of Weitzman's partial equilibrium framework, into a single-period model of Mechanisms for Abating Global Emissions under Stochasticity (MAGES), which includes:

- exogenous uncertainties in both BAU emissions and MACs;
- many emitting parties, which can be firms or countries, depending on whether pollutant is national or truly global;
- a primary choice of revenue-neutral mechanism type among ET, a tax with thresholds and NTTs (unsurprisingly, NTTs will be welfare-

1. Where parties are countries, whatever their realised financial loss or gain from a tax with thresholds, the country can choose how it distributes losses and gains among producers and consumers. So the valid distributional concern that GHG control will create monopoly profits for carbon producers, raised by Pezzey and Park (1998) and quantified by Bovenberg and Goulder (2001), which suggests using a non-revenue-neutral mechanism at country level, is a separate issue.

dominated by ET, but we include it here as an important point of comparison to single-party analyses like Weitzman);

- a secondary choice of the degree to which the mechanisms targets or thresholds are indexed to parties' activity levels (e.g. GDP for countries), including optimal intensity targets (with indexation optimised individually) and absolute targets (no indexation) as special cases;
- risk aversion and hence endogenous targets, though for completeness we also consider risk neutrality for each mechanism.

The MAGES model abstracts from other features in order to be able to encompass the five above. Dynamics and discounting are absent: we consider only a one-shot equilibrium some time in the future, and so for example ignore all problems of the lumpiness and irreversibility of investments in abatement. We also assume perfect enforcement. As an empirical application of MAGES, we study global GHG abatement in a world of 18 regions in 2020, using a new numerical simulation model (MAGES-GHG) containing many new parameter estimates. As in Nordhaus and Yang (1996), our model gives regional results, but these and most calibration details are given in another paper (Jotzo and Pezzey 2006, hereafter JP) with a primarily empirical focus.

The paper is laid out as follows. Section 2 gives full theoretical details of the MAGES model, and results for expected global net benefit when this is the welfare measure that optimal policy chooses to maximise, under the assumption that all parties are risk-neutral. We also give empirical results for global GHG abatement in 2020, using MAGES-GHG. In Section 3 we give theoretical, risk-averse results for expected global payoff when this is the welfare measure maximised, subject to an equity criterion; and again give empirical results. Section 4 concludes.

2. THE MODEL, AND RESULTS UNDER RISK NEUTRALITY

Here we build our multi-party, multi-uncertainty, flexible-commitment generalisation of Weitzman's classic partial equilibrium model. Section 2.1 defines the three uncertainties in BAU emissions, and general flexible targets, which enables Section 2.2 to define how emissions trading (ET), a tax with thresholds and non-tradable targets (NTTs) operate as abatement mechanisms. Section 2.3 gives basic definitions of (risk-neutral) net benefit and (risk-averse) 'payoff', and Section 2.4 defines each party's benefit and cost functions which go into net benefit. Section 2.5 gives the key results for expected global net benefit under each mechanism, and Section 2.6 gives the corresponding result for unilateral abatement by all parties. Section 2.7 presents empirical results for the expected global gain in net benefit and global abatement under each mechanism for GHG abatement in 2020.

The MAGES model applies to any situation where many emitting parties are small, and 'global' emissions (that is, total emissions by all parties) are well-mixed in some environmental reservoir, which may be the whole world or just part of it. Each party being 'small' means just that it takes any global endogenous variables, such as emissions or any global permit price, as given. The parties are indexed either by $i = 1,...,n$ or $k = 1,...,n$, and these summation notations are used throughout, for any parameter or variable J:

$$\Sigma_{i=1}^{n} J_i \equiv \Sigma J_i \equiv \Sigma J_k \equiv J; \quad \Sigma_{i=1}^{n-1} J_i \equiv \Sigma_{-n} J_i; \quad \{J_i\} \equiv (J_1,...,J_n) \equiv \mathbf{J}. \qquad [2.1]$$

Another notation used everywhere is that a tilde ($\tilde{}$) denotes a random variable, and the same variable without a tilde is its expectation:

$$J := \mathrm{E}[\tilde{J}]. \qquad [2.2]$$

[Referees' Appendix 1 lists the large notation set needed for MAGES.]

All MAGES variables apply to some unspecified time in the 'future', far enough away for some variables to be significantly stochastic; and the type of abatement mechanism, and the target levels and degrees of indexation in that mechanism, must be chosen 'now' while stochasticity remains. All costs and benefits are undiscounted, since they all occur at the same future time, so there would be no purpose in discounting them from the future to now, and to save repetition, the 'future' qualifier is generally omitted.

2.1 BAU emissions, and emission targets/thresholds

A party's realised (uncertain), BAU emissions are denoted $\tilde{E}^b_i$ t/yr. Its abatement is generally denoted $\tilde{Q}_i$ t/yr (superscripts may be added later to denote specific abatement policies).[2] Its abated emissions are then

$$\tilde{E}_i := \tilde{E}^b_i - \tilde{Q}_i \text{ t/yr.} \qquad [2.3]$$

We consider three random variables as sources of uncertainty in party i's BAU emissions:

- ε_{Yi}, the proportional random variation in party i's overall *activity level* $\tilde{Y}_i$ (such as product output for a firm, or GDP for a country), to which a fraction α_i of expected BAU emissions, E^b_i, are linked;
- $\varepsilon_{\eta i}$, representing uncertainty in party i's *intensity* (emissions/activity ratio) $\tilde{\eta}_i$;
- $\varepsilon_{\rho i}$, representing uncertainty in a $(1-\alpha_i)$ fraction of expected BAU emissions, which have no link to the activity level.

As explained in more detail in JP, the net effect of these three uncertainties is that i's BAU emissions are

$$\tilde{E}^b_i \approx [1+\alpha_i(\varepsilon_{Yi}+\varepsilon_{\eta i})+(1-\alpha_i)\varepsilon_{\rho i}]\, E^b_i \text{ t/yr.} \qquad [2.4]$$

---

2. Typical units are shown only where they first appear, or for clarification.

Importantly, we assume that ε_{Yi}, $\varepsilon_{\eta i}$ and $\varepsilon_{\rho i}$ are three sets of random errors, all independent of all the others, and with zero means:[3]

$$E[\varepsilon_{Yi}] = E[\varepsilon_{\eta i}] = E[\varepsilon_{\rho i}] = E[\varepsilon_{Yi}\varepsilon_{\eta k}] = E[\varepsilon_{\eta i}\varepsilon_{\rho k}], \text{ etc} = 0 \quad \forall\ i,k \qquad [2.6]$$

$$E[\varepsilon_{Yi}^2] =: \sigma_{Yi}^2;\ \ E[\varepsilon_{\eta i}^2] = \sigma_{\eta i}^2;\ \ E[\varepsilon_{\rho i}^2] = \sigma_{\rho i}^2 \quad \forall\ i,k \qquad [2.7]$$

Next, we define party i's level of financial commitment to abatement as a general *flexible emissions target (or threshold, for a tax)*

$$\tilde{X}_i = X_i(1+\beta_i\varepsilon_{Yi}) \text{ t/yr}, \quad \beta_i \geq 0, \quad \text{hence from [2.6]}, \qquad [2.8]$$

$$x_i = X_i/E^b_i, \quad \text{hence} \quad \tilde{X}_i = x_iE^b_i(1+\beta_i\varepsilon_{Yi}). \qquad [2.9]$$

Here X_i is the expected target, β_i is the degree to which the realised target $\tilde{X}_i$ is indexed to activity, and x_i is the expected target as a proportion of BAU emissions. We focus on two special cases, of which the second appears to be new:

$\beta_i = 0\ \forall i \quad \Rightarrow$ *absolute* target (i.e. no flexibility),

$$\tilde{X}_i = x_iE^b_i = X_i; \qquad [2.11]$$

$\beta_i = \alpha_i/x_i\ \forall i \quad \Rightarrow$ *(optimal) intensity* target, $\tilde{X}_i = x_iE^b_i[1+(\alpha_i/x_i)\varepsilon_{Yi}]$. [2.13]

The reason for calling the latter 'optimal' will become clear soon.[4]

We often need the difference between party i's BAU emissions and its target/threshold, and from [2.9] and [2.4] it is:

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3. In principle, MAGES can handle non-independence, but we would then need to include theoretically, and estimate empirically, a matrix of $(3n-1)(3n/2)$ covariances.

4. A third special case, considered at length in JP, is $\beta_i = 1\ \ \forall\ i$, known as *standard (or one-to-one) intensity* targets, $\tilde{X}_i = x_iE^b_i(1+\varepsilon_{Yi})$. Because this is much the more common definition of intensity targets, we repeat our 'optimal' qualifier occasionally.

$\tilde{E}^b{}_i - \tilde{X}_i \quad = E^b{}_i - X_i + \tilde{N}_{Ei}$, hence $\tilde{E}^b - \tilde{X} = E^b - X + \tilde{N}_E$; [2.14]

where a party's *net emissions uncertainty* (net of any neutralising effect of target indexing by β_i) is

$$\tilde{N}_{Ei} := [(\alpha_i - \beta_i x_i)\varepsilon_{Yi} + \alpha_i\varepsilon_{\eta i} + (1-\alpha_i)\varepsilon_{\rho i}]\ E^b{}_i. \qquad [2.16]$$

From [2.6] and [2.7], *expected squared net emissions uncertainty* is then:

$$D_{Ei} := \mathrm{E}[\tilde{N}_{Ei}{}^2] = [(\alpha_i - \beta_i x_i)^2\sigma_{Yi}{}^2 + \alpha_i{}^2\sigma_{\eta i}{}^2 + (1-\alpha_i)^2\sigma_{\rho i}{}^2]\ E^b{}_i{}^2, \qquad [2.17]$$

with global expected squared net emissions uncertainty, and expected global squared net emission uncertainty being both:

$$D_E \quad := \mathrm{E}[\Sigma\tilde{N}_{Ei}{}^2] = \Sigma(\mathrm{E}[\tilde{N}_{Ei}{}^2]) = \mathrm{E}[(\tilde{N}_E)^2] \qquad [2.18]$$

$$= \Sigma\{[(\alpha_i - \beta_i x_i)^2\sigma_{Yi}{}^2 + \alpha_i{}^2\sigma_{\eta i}{}^2 + (1-\alpha_i)^2\sigma_{\rho i}{}^2]\ E^b{}_i{}^2\} \qquad [2.19]$$

The formulae for the special cases in [2.11] and [2.13] are needed later. For absolute targets, denoted by ^{0}:

$$\tilde{N}^0{}_{Ei} \ := [\alpha_i\varepsilon_{Yi} + \alpha_i\varepsilon_{\eta i} + (1-\alpha_i)\varepsilon_{\rho i}]\ E^b{}_i, \text{ and} \qquad [2.21]$$

$$D^0{}_{Ei} \ := \mathrm{E}[\tilde{N}^0{}_{Ei}{}^2] = [\alpha_i{}^2\sigma_{Yi}{}^2 + \alpha_i{}^2\sigma_{\eta i}{}^2 + (1-\alpha_i)^2\sigma_{\rho i}{}^2]\ E^b{}_i{}^2; \qquad [2.22]$$

while for intensity targets, denoted by *:

$$\tilde{N}^*{}_{Ei} \ := [\alpha_i\varepsilon_{\eta i} + (1-\alpha_i)\varepsilon_{\rho i}]\ E^b{}_i, \text{ and} \qquad [2.23]$$

$$D^*{}_{Ei} \ := \mathrm{E}[\tilde{N}^*{}_{Ei}{}^2] = [\alpha_i{}^2\sigma_{\eta i}{}^2 + (1-\alpha_i)^2\sigma_{\rho i}{}^2]\ E^b{}_i{}^2. \qquad [2.24]$$

2.2 Abatement mechanisms

Using the above notations, our chosen control mechanisms, as laid down by some national policy or international treaty (we mostly use 'policy') are:

(a) *Emissions Trading* (ET), denoted ^{T}. We assume the policy defines a set of expected future targets $\{X^T_i\}$ (with $X^T < E^b$, to make expected abatement positive) and indexes $\{\beta_i\}$, which are combined using [2.8] into a set of flexible realised targets, $\{\tilde{X}^T_i\}$, given freely (grandfathered) to *i*. Because targets are tradable, perfect enforcement makes abated emissions equal the target only at the global level:

$$\tilde{E}^T = \tilde{X}^T \text{ t/yr.} \tag{2.26}$$

A party's (realised) *emission trading revenue* (possibly negative) is its target $\tilde{X}^T_i$ minus its abated emissions $\tilde{E}^T_i$, times the emission price attained by the global permit market, denoted $\tilde{p}^T$ \$/t (which determines abatement $\tilde{Q}^T_i$, and is itself determined later). So from [2.3]:

$$\tilde{R}^T_i := \tilde{p}^T[\tilde{X}^T_i - \tilde{E}^b_i + \tilde{Q}^T_i(\tilde{p}^T)], \tag{2.27}$$

and global revenue-neutrality is automatic ($\tilde{R}^T \equiv 0$ from [2.26] and [2.3]).

(b) An emissions *tax with thresholds* ('control by price'), denoted $^\$$ and often shortened to just 'a tax'. Here the policy defines flexible tax thresholds $\{\tilde{X}^\$_i\}$ (with $X^\$ < E^b$) as in [2.8], and sets a global, certain emissions tax

$$p^\$ = (E^b - X^\$)/M \text{ \$/t.} \tag{2.28}$$

Thresholds are akin to property rights like grandfathered tradable targets, and each party receives a gross tax revenue *from* the policy authority of

$$\tilde{K}^\$_i := p^\$[\tilde{X}^\$_i - \tilde{E}^b_i + \tilde{Q}^\$_i(p^\$)] \text{ \$/yr.} \tag{2.29}$$

In a certainty model, that is with $K^\$_i := p^\$(X^\$_i - E^\$_i)$, the role of the thresholds $X^\$_i$ originated in Mumy's (1980) 'property rights sharing', and was taken up by Pezzey (1992) and Quirion (2004) as 'baseline effluent rights', and by Farrow (1995) as 'tax credits'. Our first contribution is to allow thresholds to be flexible rather than fixed, for symmetry with the other mechanisms rather than as a practical proposal for policy.

Whether or not thresholds are flexible, uncertainty has an important effect on (global) revenue-neutrality. The authority is assumed to know the expected parameters in [2.28] and thus how to set $p^\$$ so that expected total emissions $E^\$(p^\$)$ equal expected total thresholds $X^\$$ and thus make *expected* revenue zero. (This does not have to happen, of course; Pezzey (1992) stresses that $X^\$$ is a political choice, that should be free to lie anywhere between 0 and $E^\$(p^\$)$.) But it cannot make realised total emissions $\tilde{E}^\$$ equal realised total thresholds $\tilde{X}^\$$ and thus *realised* revenue zero. Its realised gross payment to all parties is:

$$\tilde{K}^\$ = p^\$[\tilde{X}^\$-\tilde{E}^\$(p^\$)] \neq 0, \text{ even though } K^\$ = p^\$[X^\$-E^\$(p^\$)] \equiv 0. \quad [2.31]$$

Our second contribution to the tax-with-thresholds mechanism is therefore to introduce a *clawback* to make global revenue zero in every realised future, whereby the authority claws back a tax adjustment of $(X^\$_i/X^\$)\tilde{K}^\$$ each party. This is effectively a lump sum beyond i's influence which makes i's net tax revenue equal to:

$$\tilde{R}^\$_i := \tilde{K}^\$_i - (X^\$_i/X^\$)\tilde{K}^\$, \text{ with global realised neutrality, } \tilde{R}^\$ \equiv 0. \quad [2.32]$$

(c) *Non-Tradable Targets* (NTTs, 'control by quantities'), denoted $^@$. Here the authority mandates (and perfectly enforces) that each party i's abated emissions equal its flexible target $\tilde{X}^@_i$ defined by [2.8] (with $X^@ < E^b$):

$$\tilde{E}^@_i = \tilde{X}^@_i \text{ t/yr, with obviously no trading revenue, } \tilde{R}^@_i \equiv 0. \quad [2.33]$$

For definitions and results common to all three abatement policy mechanisms, we use ^{j} as an index, so $^j = {}^T, {}^\$, {}^@$. We use a second superscript ^{0} or * to show when targets/thresholds are absolute or (optimal) intensity, respectively.

2.3 Unilateral vs. policy abatement, and net benefit and payoff

We focus on three ‘future’, symmetric equilibrium states of the world, each with a condition determining (future, uncertain) emissions abatement:

(i) *No abatement*, so that all parties have BAU emissions;

(ii) All parties abate collectively in accordance with a *policy*, already defined as a government policy for all emitting parties (usually firms) in one country, or an international treaty signed by all emitting countries (or groups of countries) on the globe. A policy is defined by its choices of mechanism type (ET vs tax vs NTT), target or threshold levels $\{X_i\}$ and indexing $\{\beta_i\}$, and (for the tax only) the emission tax rate, $p^\$$.

(iii) All parties abate *unilaterally*, with variables in this state denoted ^{U}. This differs from no abatement, since any party that cares about global abatement will do some abatement on its own, albeit much less than under a policy.

These three states suffice for our analysis of cooperative (or within a nation, compulsory) policy formation. For non-cooperative analyses, other, asymmetric states − such as only some parties joining a treaty while others abate unilaterally (i.e. free-ride on the treaty) − would obviously be interesting too, but these remain for further work.

Two alternative measures are available in MAGES to compare the desirability of policy-based abatement both among mechanisms, and with unilateral abatement:

− Party i’s *net benefit* from abatement policy j compared to no abatement is $\tilde{A}^j_i$ \$/yr (its ‘*A*dvantage’). This is its dollar-valued benefit $\tilde{B}^j_i$ of global abatement $\tilde{Q}^j$, including its trading or tax revenue (if any) under the policy, minus the cost $\tilde{C}^j_i$ of its own abatement $\tilde{Q}^j_i$:

$$\tilde{A}^j_i := \tilde{B}^j_i - \tilde{C}^j_i. \qquad [2.34]$$

[2.34] also applies to the unilateral state, i.e. $\tilde{A}^U_i := \tilde{B}^U_i - \tilde{C}^U_i$. This assumes that the net benefit a party perceives from the introduction of an abatement policy is framed solely in terms of the financial and environmental consequences of the policy. We discuss this important framing effect further in JP. A quite different caveat is that In a second-best world, [2.34] should be amended to allow for the marginal cost of public funds being greater than unity (Quirion 2004), but this remains for further work. What in fact we report later is the *gain* in net benefit by moving from unilateral to abatement using mechanism *j*:

$$\tilde{G}^j_i := \tilde{A}^j_i - \tilde{A}^U_i. \qquad [2.35]$$

− Party *i*'s *payoff* from moving from unilateral to abatement mechanism *j* is defined as the gain in net benefit between the two states, plus a strictly concave function of the difference in net benefit:

$$\tilde{U}^j_i(\tilde{G}^j_i) := \tilde{G}^j_i + z_i(1-e^{-r\tilde{G}^j_i}) \ \text{\$/yr}; \ \ z_i \ \text{\$/yr} > 0, \ \ r \ \text{yr/\$} > 0. \qquad [2.36]$$

The strictly concave form of payoff makes parties risk averse, and we feel maximising payoff should give a more realistic guide than maximising net benefit to parties' decisions about abatement policy proposals. (The *U* notation for payoff in [2.36] is suggestive of 'utility' and hence a diminishing marginal utility of income, and is not to be confused with the ^{U} for unilateralism in [2.35].) So in Section 3 we focus on expected global payoff, $E[\Sigma\tilde{U}^j_i]$, as a measure of the desirability of abatement policy *j*. Formula [2.36] assumes that cross-party variations in risk aversion can be adequately captured by variations in the scaling parameters $\{z_i\}$, leaving the curvature parameter r as a global constant, and that is what we consider most justified in the empirical GHG case. However, the algebra is quite workable with a set of curvature parameters, $\{r_i\}$, instead.

Despite our ultimate preference for risk aversion, we give results in this Section for the risk-neutral case, where the optimal policy is to choose emission targets/thresholds and (perhaps) an emission price to *maximise expected global net benefits*. We can then make interesting comparisons of our results to those in Weitzman (1974), who assumed risk neutrality. And to clarify these comparisons, we highlight subcases of abatement mechanisms where the distribution of targets among parties within a given total has no effect on expected global net benefit, and so remains indeterminate under risk neutrality.

2.4 Abatement benefits and costs

We assume that dollar-valued benefit that party i gets from abatement mechanism j giving global (well-mixed) abatement $\tilde{Q}^j$ is:

$$\tilde{B}^j{}_i(\tilde{Q}^j) \quad := V_i\tilde{Q}^j - ½W_i(\tilde{Q}^j)^2; \quad V_i \;\; \$/\mathrm{t} > 0, \;\; W_i \;\; \$.\mathrm{yr/t}^2 > 0. \tag{2.38}$$

hence global benefit

$$\tilde{B}(\tilde{Q}^j) \quad := V\tilde{Q}^j - ½W(\tilde{Q}^j)^2 \quad \$/\mathrm{yr} \tag{2.39}$$

Calibration of the benefit parameters $\{V_i\}$ and $\{W_i\}$ in the GHG case is done by inference, so as to produce plausible global abatement in our *Reference Case*, defined as using emissions trading with absolute targets when risk aversion is taken into consideration (which is done only in Section 3).

For i's abatement costs (net of emissions trading), we assume that:

$$\tilde{C}^j_i(\tilde{Q}^j_i) = ½(1/M_i)(\tilde{Q}^j_i)^2 + \tilde{Q}^j_i\varepsilon_{Ci} - \tilde{R}^j_i \text{ \$/yr}$$

$$= ½(1/M_i)(\tilde{Q}^j_i)^2 + (1/M_i)\tilde{Q}^j_i\tilde{N}_{Ci} - \tilde{R}^j_i, \text{ where} \qquad [2.41]$$

$$\tilde{N}_{Ci} := M_i\varepsilon_{Ci}, \ \mathrm{E}[\varepsilon_{Ci}] = 0; \ \mathrm{E}[\varepsilon_{Ci}^2] =: \sigma_{Ci}^2; \text{ and hence} \qquad [2.42]$$

$$\tilde{C}^j_i{}'(\tilde{Q}^j_i) = (1/M_i)\tilde{Q}^j_i + \varepsilon_{Ci} = (1/M_i)\tilde{Q}^j_i + (1/M_i)\tilde{N}_{Ci}. \qquad [2.43]$$

Parameter M_i t²/\$.yr is called an *abatement potential*, and $1/M_i$ is the slope of i's MAC curve. As in Weitzman (1974), we call ε_{Ci} the *shift uncertainty in MAC*, and unlike his full analysis (as noted by Stavins 1996), we assume

ε_{Ci} is independent of all other uncertainties (ε_{Ck}, ε_{Yi}, $\varepsilon_{\eta k}$, etc), [2.44]

so if we also define

$$D_{Ci} := \mathrm{E}[\tilde{N}_{Ci}^2] = M_i^2\sigma_{Ci}^2, \text{ then } D_C := \Sigma M_i^2\sigma_{Ci}^2 = \mathrm{E}[\tilde{N}_C^2]. \qquad [2.46]$$

Both the calibration of $\{M_i\}$ for the 2020 GHG case, and a discussion of the plausibility of this independence assumption in that case, are given in JP. We have left a shift uncertainty out of the benefit function [2.38] because we found, after Weitzman (1974), that such uncertainties had no effect on any risk-neutral results (though could perhaps have a minor effect on expected payoff), as long as they are uncorrelated with the cost uncertainties introduced next. As noted by Stavins (1996), this may then limit the practical applicability of our results; but such correlation is negligible in the greenhouse case used here as an empirical case study.

Equations [2.34], [2.38] and [2.41] together mean that i's realised net benefit from mechanism j is

$$\tilde{A}^j_i = V_i\tilde{Q}^j - ½W_i(\tilde{Q}^j)^2 - ½(1/M_i)(\tilde{Q}^j_i)^2 - (1/M_i)\tilde{Q}^j_i\tilde{N}_{Ci} + \tilde{R}^j_i \qquad [2.48]$$

2.5 Expected global net benefit under policy abatement

We now give formulae for the expected global net benefit from using each abatement mechanism under a risk-neutral policy objective of maximising expected *global* net benefit. The computation of each formula starts with some way of choosing each party's abatement so that its MAC equals some global emission price or shadow price (not so that individual net benefit $\tilde{A}^j_i$ is maximised). However, the ways for ET and a tax both entail individual agents in all parties using their market freedom to choose their abatement and hence their abated emissions, so as to maximise their profits by equating their *realised* individual MACs with the global permit price or tax.

No such freedom is allowed with non-tradable targets (NTTs), where the authority mandates parties' abated emissions to equal targets, which it chooses so that parties' *expected* MACs are all the same, shadow price, so as to achieve any given global abatement at minimum expected total cost. Though under the general rule [2.8], $\tilde{X}^{@}_i$ is flexible in responding to *i*'s emissions uncertainty, it does not respond to *i* having unexpectedly high or low MAC, and this is the source of NTTs' relative inefficiency.

So with *ET*, MAC ($\tilde{C}_i'$ from [2.43]) is equated to the global permit price:

$$(1/M_i)\tilde{Q}^T_i + (1/M_i)\tilde{N}_{Ci} = \tilde{p}^T,$$

which on rearranging gives

$$\tilde{Q}^T_i = \tilde{p}^T M_i - \tilde{N}_{Ci}, \text{ hence } \tilde{Q}^T = \tilde{p}^T M - \tilde{N}_C = \tilde{E}^b - \tilde{X}^T, \qquad [2.49]$$

which on using [2.4] gives

$$\tilde{p}^T = (E^b - X^T + \tilde{N}_E + \tilde{N}_C)/M, \text{ with expectation} \qquad [2.50]$$

$p^T \quad = (E^b - X^T)/M$ (> 0 by choice of X^T, so some abatement occurs). [2.51][5]

Hence $\tilde{p}^T = p^T + (\tilde{N}_E + \tilde{N}_C)/M$, which with [2.49] gives

$$\tilde{Q}^T = p^T M + \tilde{N}_E = E^b - X^T + \tilde{N}_E. \qquad [2.52]$$

Three observations apply to these intermediate results. First, the linearity of MAC [2.43] means that a choke price E^b_i/M_i exists, at which expected, abated emissions are zero. In practice zero emissions may be impossible, meaning there are limits beyond which the quadratic cost function [2.41] is unacceptably inaccurate − a caution which applies to all three mechanisms. Second, [2.50] means that under ET, a party's trading revenue is affected not just by its own uncertainties, but also by all other parties' uncertainties $\tilde{N}_E + \tilde{N}_C$, transmitted through the global price $\tilde{p}^T$. Third, [2.50] also shows that even though $p^T > 0$ because $X^T < E^b$ by assumption, extreme realisations can have $\tilde{p}^T < 0$ (hence no emissions trading). In our GHG empirical case, the latter occurs only about once in a thousand realisations, so we ignore it.

With the *tax*, MAC is equated with the (certain) emissions tax $p^\$$:

$$(1/M_i)(\tilde{Q}^\$_i + \tilde{N}_{Ci}) = p^\$, \text{ hence} \qquad [2.53]$$

$$\tilde{Q}^\$_i = p^\$ M_i - \tilde{N}_{Ci} \text{ and } \tilde{Q}^\$ = p^\$ M - \tilde{N}_C; \text{ hence} \qquad [2.54]$$

$$p^\$ = (E^b - X^\$)/M, \text{ again assumed} > 0 \text{ for abatement to occur.} \qquad [2.55]$$

5. In theory, the realised permit price $\tilde{p}$ in [2.50] could be negative in some realisations, which would imply a subsidy on emissions or a penalty for holding emissions permits. However, this is of no empirical relevance in the modelling, as occurs in 0.25% of random realisations at most in the scenarios presented in this paper.

With *NTTs*, the authority chooses each non-tradable target $X^{@}_i$ so that i's *expected* MAC equals a common shadow price, say $p^{@}$ \$/t:

$$(1/M_i)Q^{@}_i = p^{@} \quad \forall\ i, \ \text{ for some } p^{@} > 0; \ \text{ hence} \qquad [2.56]$$

$$Q^{@}_i = E^b_i - X^{@}i = p^{@}M_i \ \text{ and } \ Q^{@} = p^{@}M = E^b - X^{@}; \ \text{ hence} \qquad [2.57]$$

$$p^{@} = (E^b - X^{@})/M. \qquad [2.58]$$

Then from [2.33], [2.4] and [2.57], i's realised abatement is:

$$\tilde{Q}^{@}_i = \tilde{E}^b_i - \tilde{X}^{@}_i = E^b_i - X^{@}_i + \tilde{N}_{Ei} = p^{@}M_i + \tilde{N}_{Ei} \qquad [2.59]$$

Appendices 1-3 then establish the following risk-neutral, expected global net benefits[6] for the three mechanisms (where the D_{Ci} term in the tax result has been aligned to make comparison with the ET result easier):

Proposition 1: Optimised expected global net benefits from a global abatement policy instead of no abatement are

for emissions trading (ET) with flexible targets:

$$A^T = \bar{A}^T - \tfrac{1}{2}\Sigma(1/M+W)D_{Ei}(x^T_i) + \tfrac{1}{2}\Sigma(1/M_i - 1/M)D_{Ci}; \qquad [2.60]$$

for a tax with flexible thresholds:

$$A^{\$} = \bar{A}^{\$} + \tfrac{1}{2}\Sigma(1/M_i - W)D_{Ci}; \qquad [2.61]$$

for non-tradable, flexible targets (NTTs):

$$A^{@} = \bar{A}^{@} - \tfrac{1}{2}\Sigma(1/M_i+W)D_{Ei}(x^{@}_i); \qquad [2.62]$$

where

$$\bar{A}^j(p^j) := p^jVM - \tfrac{1}{2}M(1+WM)(p^j)^2; \ \ p^j(X) := (E^b - X^j)/M; \ {}^j = {}^T, {}^{\$}, {}^{@}. \qquad [2.63]$$

6. Each Appendix first calculates party i's realised net benefit, $\tilde{A}^j_i$, $^j = {}^T, {}^{\$}, {}^{@}$ in turn, and then the corresponding global expectation A. However, the results for $\tilde{A}^j_i$ itself matter only when expected payoff is maximised, so we report them in Section 3.

One obvious feature of these results is that the tax eliminates emissions uncertainty, and NTTs eliminate MAC uncertainty, while ET eliminates neither. This stems directly from the different ways (control by 'tradable quantities', 'price' or 'quantities') in which the mechanisms operate. Also obvious from [2.60] and [2.61] is:

Corollary 1: (a) Under emissions trading, Marginal Abatement Cost uncertainty increases expected global net benefit compared to certainty; (b) under a tax, MAC uncertainty increases expected global net benefit compared to certainty, provided the slope of the global marginal benefit curve is lower than a weighted mean slope of the marginal cost curves.

The increase under (a), $\frac{1}{2}\Sigma(1/M_i - 1/M)D_{Ci}$, is positive whenever $M = \Sigma M_k > M_i$, i.e. in any multi-party world. (Typically, $1/M_i >> 1/M$, and for MAGES-GHG with $n = 18$, $M > 4$ [$\max_i\{M_i\}$], which is for China.) Intuitively, if all parties take the global emission price as given (as with ET and a tax, but not NTTs), any deviation in a party's MAC gives a decrease or increase of producer surplus proportional to the deviation squared, and hence a positive mean increase. A simple algebraic illustration is the expected surplus from choosing an equal chance of quantities $q+\varepsilon$ or $q-\varepsilon$, instead of a certain q, when surplus is proportional to quantity squared, since $\frac{1}{2}[(q+\varepsilon)^2+(q-\varepsilon)^2] - q^2 = \varepsilon^2 > 0$. The single-party equivalent of effect (b), where $\frac{1}{2}\Sigma(1/M_i - W)D_{Ci} > 0$ whenever $W < [\Sigma(1/M_i)D_{Ci}]/D_C$, was present but unremarked in Weitzman (1974). The insight that MAC uncertainty improves the attractiveness of both ET and a tax appears to have been overlooked so far.

More precise comparisons of the three mechanisms are hampered whenever the D_{Ei} terms depend on $\{x_i\}$. So let us consider only the cases of absolute or (optimal) intensity targets (hence the D_{Ei}'s have no dependence on x_i's, from [2.22] and [2.24]), or of no emissions uncertainty (hence all $D_{Ei} = 0$). In such cases, the only effect of target choice on net benefit is the effect of the total X on the emission price p, and hence on $\bar{A}[p]$. The optimal policy then entails an indeterminate target distribution $\{x_i\}$, and the *same* optimal price (say $\bar{p}$) for all three mechanisms. The latter comes from setting $\bar{A}'[p] = VM - M(1+WM)\bar{p} = 0$, and makes expected global abatement, target total, benefit, cost and net benefit also the same for all three mechanisms, namely:

Proposition 2: Risk-neutral, global results when target distribution is indeterminate, for all abatement mechanisms (ET ^{T}, tax $^\$$ or NTTs $^@$):

$$\begin{aligned}
&\textit{price} && = \bar{p} := V/(1+WM) \ \ \$/\text{t};^{7} && \Big) \quad [2.64]\\
&\textit{abatement} && = \bar{Q} := \bar{p}M \ \ \text{t/yr}; && \Big)\\
&\textit{target total} && = \bar{X} := E^b - \bar{p}M \ \ \text{t/yr}; && \Big)
\end{aligned}$$

and certainty components (i.e. ignoring all D-terms) of benefit and cost are

$$\begin{aligned}
&\textit{benefit} && = \bar{B} := \tfrac{1}{2}\bar{p}M(\bar{p}+V) \ \ \$/\text{yr}; && \Big)\\
&\textit{cost} && = \bar{C} := \tfrac{1}{2}\bar{p}^2M \ \ \$/\text{yr}; && \Big)\\
&\textit{net benefit} && = \bar{A} := \tfrac{1}{2}\bar{p}VM \ \ \$/\text{yr}. && \Big)
\end{aligned}$$

So in Proposition 1 we now have $\bar{A}^T = \bar{A}^\$ = \bar{A}^@ = \bar{A}$, and the only differences among mechanisms are in the net costs of uncertainty (the D_{Ei} and D_{Ci} terms). Now consider emissions uncertainty again, but confined again to absolute or intensity targets so that Proposition 2 applies. We then immediately have the following five results which are easy to compare

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7. Note how, from [2.39] and the expectation of [2.49], [2.54] or [2.57], $\bar{p}$ obeys the Samuelson (1954) prescription for the optimal price of a public good under certainty, namely $\bar{p} = B'(\bar{Q}) = V - W\bar{Q} = V - W\bar{p}M$.

directly (using [2.22] and [2.24] to compute D^0_{Ei} and D^*_{Ei}):

Proposition 3: Risk-neutral, global expected benefits from abatement when target distribution is indeterminate are,

for emissions trading (ET) with absolute targets:

$$A^{T0} = \tfrac{1}{2}\bar{p}VM - \tfrac{1}{2}\Sigma(1/M+W)D^0_{Ei} + \tfrac{1}{2}\Sigma(1/M_i-1/M)D_{Ci}; \qquad [2.65]$$

for emissions trading (ET) with (optimal) intensity targets:

$$A^{T*} = \tfrac{1}{2}\bar{p}VM - \tfrac{1}{2}\Sigma(1/M+W)D^*_{Ei} + \tfrac{1}{2}\Sigma(1/M_i-1/M)D_{Ci}; \qquad [2.66]$$

for a tax with general flexible thresholds:

$$A^{\$} = \tfrac{1}{2}\bar{p}VM + \tfrac{1}{2}\Sigma(1/M_i-W)D_{Ci}; \qquad [2.67]$$

for non-tradable, absolute targets (NTTs):

$$A^{@0} = \tfrac{1}{2}\bar{p}VM - \tfrac{1}{2}\Sigma(1/M_i+W)D^0_{Ei}. \qquad [2.68]$$

for non-tradable, (optimal) intensity targets (NTTs):

$$A^{@*} = \tfrac{1}{2}\bar{p}VM - \tfrac{1}{2}\Sigma(1/M_i+W)D^*_{Ei}. \qquad [2.69]$$

In comparing one mechanism to another, perhaps the most striking point is the loss of Weitzman's simple rule for choosing prices over quantities, now that both emissions and MAC are uncertain. A higher global marginal benefit, W, reduces net benefit from price control because of the $-\tfrac{1}{2}WD_C$ in [2.68], but it also reduces net benefit from (tradable or non-tradable) quantity control because of the $-\tfrac{1}{2}WD_E$ in the other four results.

Again because $M >> M_i$, $A^{T0}-A^{@0} = \tfrac{1}{2}\Sigma(1/M_i-1/M)(D^0_{Ei}+D_{Ci}) >> 0$, showing large gains from making targets tradable, which stem from both emissions and cost uncertainties. Similarly, $A^{@*}-A^{@0} >> A^{T*}-A^{T0}$, showing that target flexibility increases net benefit by much more if targets are non-tradable than if they are tradable. In Section 2.7 we will report empirical results for ET and NTTs only for the above five cases.

2.6 Expected global net benefit under unilateral abatement

We give unilateral results here for use later. A simple variant of [2.48] also applies to the unilateral case, namely that net benefit is

$$\tilde{A}^U{}_i := V_i\tilde{Q}^U - ½W_i(\tilde{Q}^U)^2 - ½(1/M_i)(\tilde{Q}^U{}_i)^2 - (1/M_i)\tilde{Q}^U{}_i\tilde{N}_{Ci}. \qquad [2.70]$$

Individually optimal abatement $\tilde{Q}^U{}_i$ is chosen to maximise $\tilde{A}^U{}_i$ on the non-cooperative assumption that $\partial\tilde{Q}^U/\partial\tilde{Q}^U{}_i = 1$, giving (see Appendix 4)

$$\tilde{Q}^U = \Sigma(V_iM_i - \tilde{N}_{Ci}) / (1+\Sigma W_kM_k), \text{ hence} \qquad [2.71]$$

$$Q^U = \Sigma V_iM_i / (1+\Sigma W_kM_k); \text{ and} \qquad [2.72]$$

$$\tilde{A}^U{}_i := V_i\tilde{Q}^U - ½W_i(\tilde{Q}^U)^2 - ½(1/M_i)(V_iM_i - \tilde{N}_{Ci} - W_iM_i\tilde{Q}^U)^2 - (1/M_i)(V_iM_i - \tilde{N}_{Ci} - W_iM_i\tilde{Q}^U)\tilde{N}_{Ci}, \text{ hence} \qquad [2.73]$$

$$A^U{}_i = \bar{A}^U{}_i + F^U{}_i \text{ where} \qquad [2.74]$$

$$\bar{A}^U{}_i := V_iQ^U - ½W_i(1+W_iM_i)(Q^U)^2 - ½V_i^2M_i + V_iW_iM_iQ^U, \text{ and} \qquad [2.76]$$

$$F^U{}_i = -½W_i(1+W_iM_i)[D_C/(1+\Sigma W_kM_k)^2] + ½(1/M_i)D_{Ci}. \qquad [2.77]$$

2.7 Empirical, risk-neutral results in GHG case

Here we give key results from combining the above, risk-neutral MAGES formulae with the parameter values and program of the MAGES-GHG numerical model, to show the effects of using alternative mechanisms for the case of GHG abatement in 2020. The model divides the globe into 18 regions or countries, known just as ‘countries’, ranging in GDP from Argentina and Australia to the USA and Europe. Five are high-income countries (known together as ‘the North’), while thirteen are low-income ones (‘the South’). In **Table 1** we report only the global totals or ranges of the country data we used; as already noted, the full calibration data and their

Table 1 Selected, mainly global, parameters in MAGES model of GHG abatement in an 18-country world in 2020

\$ = US\$(2000); t = tonne CO_2-equivalent; G = 10^9; T = 10^{12}

Parameter	*Calibrated value and units*
L = global population	8.2 G
Y = global GDP	88.0 T\$/yr
(E_{2000} = global emissions of GHGs in 2000	40.9 Gt/yr)
E^b = global BAU emissions of GHGs	54.0 Gt/yr (= E_{2000} × 1.32)
$(\Sigma E^b{}_i \alpha_i)/E^b$ = weighted average share of emissions linked with GDP (= energy sector share)	0.64
V = global linear valuation of abatement	21.9 \$/t
W = slope of global marginal benefit	0.219 \$.yr/G(t^2)
M = global abatement potential	0.431 G(t^2)/\$.yr
$\{\sigma_{Yi}\}$ = standard deviations of proportional uncertainty in GDP	0.13 (North), 0.18 (South)
$\{\sigma_{\eta i}\}$ = standard deviations of proportional uncertainty in intensity	0.15 (North), 0.25 (South)
$\{\sigma_{\rho i}\}$ = standard deviations of proportional emissions uncertainty outside energy sector	0.2
$\{\sigma_{Ci}\}$ = standard deviation of absolute uncertainty in MAC	4.2 \$/t (for all i)
z = risk aversion weighting parameter	1 $\$^2$/$yr^2$.person
r = worldwide risk aversion parameter	0.085 yr/\$

origins are reported in JP.

Two important features of our calibration methods are worth noting here. First, global valuation parameters V and W are *inferred* so that a global climate treaty results in a halving of 2000-2020 global emissions growth compared to BAU in our Reference Case, relative to which all percentage comparisons are reported. (A more comprehensive approach might be a survey of expert opinion in each country along the lines of Weitzman 2001, but that would take much extra work.) Second, with $y_i := Y_i/L_i$ (per capita GDP), we set each risk-aversion scaling parameter

$$z_i = 1/y_i, \qquad [2.77a]$$

as reported in the Table. The $1/y_i$ factor matches the stylised fact that uncertainty matters more in poor (low y_i) countries, but we have no further stylised facts to use to calibrate country-level risk curvature parameters $\{r_i\}$. So a global r is chosen, inferred so that global abatement in the Reference Case is *one third* lower than it would be under risk neutrality ($r = 0$).

Expected global results for risk-neutral optimisation are reported in **Table 2**. Empirical results are derived by averaging over a large number of runs where stochastic variables take on random values. MAGES is thus implemented as a stochastic, Monte Carlo simulation model. This makes it possible to impose truncations on the distributions of stochastic variables, both for more realistic representation of permit markets and for modelling of certain policy instruments. The alternative method of implementing the theoretically derived expectation formulae such as [2.74] directly in a numerical model would not allow truncations, and quickly reaches computational constraints in implementations as a multi-region model. For each scenario, the model is computed for 10,000 random realisations, with joint draws of the stochastic parameters. Realised BAU emissions are

Table 2 Results for GHG abatement mechanisms under risk-neutrality (maximisation of expected global net benefit)

(% figures are differences from the **Base Case** of Emissions Trading with Absolute targets and risk-neutrality, in contrast to the Reference Case of Section 3)

<table>
<tr><td rowspan="3">Mechanism</td><td colspan="2">Expected global gain in net benefit vs. unilateralism (G$/yr)</td><td colspan="2">Expected global abatement (Gt/yr)</td></tr>
<tr><td colspan="4">Type of thresholds or targets</td></tr>
<tr><td>Absolute</td><td>Optimal Intensity</td><td>Absolute</td><td>Optimal Intensity</td></tr>
<tr><td>Certainty case: any mechanism</td><td colspan="2">$\bar{G}(\bar{Q})$
76.1 (+14%)</td><td colspan="2" rowspan="4">$\boldsymbol{\bar{Q}}$
8.60 (0%)
(16% of E^b)
(abated emissions $\bar{E}$ = E_{2000} × 1.11)</td></tr>
<tr><td>Tax with Thresholds</td><td colspan="2">$G^{\$}(\bar{Q})$
76.1 (+14%)</td></tr>
<tr><td>Emissions Trading</td><td>$\boldsymbol{G^{T0}(\bar{Q}}$
66.8</td><td>$G^{T*}(\bar{Q})$
69.5 (+4%)</td></tr>
<tr><td>Non-Tradable Targets</td><td>$G^{@0}(\bar{Q})$
0.1 (−100%)</td><td>$G^{@*}(\bar{Q})$
29.2 (−56%)</td></tr>
<tr><td colspan="5">Base Case has price $p^{T0} = \bar{p}$ = 20 $/t, abatement benefit $\bar{B}$ = 180.5 G$/yr, cost $\bar{C}$ = 86.2 G$/yr (0.1% of Y), and net benefit $\bar{A}$ = 94.3 G$/yr (using Proposition 2). Unilateral case has abatement $\bar{Q}^U$ = 0.90 Gt/yr (1.7% of E^b), and expected net benefits of $\bar{A}^U(\bar{Q}^U)$ = 18.2 G$/yr under certainty, and $A^U(\bar{Q}^U)$ = 24.5 G$/yr under uncertainty.</td></tr>
</table>

truncated at the extreme tails of the distribution, by imposing the constraint $0.33E^b_i < \tilde{E}^b_i < 1.66E^b_i$. This truncation precludes unrealistically low or high emissions events, and applies in about 0.4% of random realisations.

The first two columns of numbers in Table 2 give the mechanism's 'gain' in expected global net benefit compared to unilateralism, $G^j = A^j - A^U$ from [2.37], rather than A, to ease comparison with the risk-averse results reported later. Given the complexity of the notation, we list these in full:

$$\left.\begin{array}{ll} \bar{G}(\bar{Q}) := \bar{A}(\bar{Q}) - \bar{A}^U; & \\ G^{\$}(\bar{Q}) := A^{\$}(\bar{Q}) - A^U; & \\ G^{T*}(\bar{Q}) := A^{T*}(\bar{Q}) - A^U, & G^{T0}(\bar{Q}) := A^{T0}(\bar{Q}) - A^U; \\ G^{@*}(\bar{Q}) := A^{@*}(\bar{Q}) - A^U, & G^{@0}(\bar{Q}) := A^{@0}(\bar{Q}) - A^U. \end{array}\right) \quad [2.78]$$

The last two columns of the table show there is no difference in global abatement among the mechanisms, so no further comment is needed there.

Consider first net benefit $A^{\$}$ in the tax case. From $W << 1/M_i$ in [2.67], we know $A^{\$}$ exceeds the certainty net benefit $\bar{A} = ½\bar{p}VM$. However, it turns out that the uncertain unilateral net benefit A^U exceeds its certainty equivalent $\bar{A}^U$ by almost exactly the same amount, making the tax and certainty gains almost the same; and the tax is clearly the best policy in principle under uncertainty. Since the tax is unfortunately off most climate policy agendas, the main question of interest is then how much closer Emissions Trading can get to the best mechanism by using intensity instead of absolute targets. The answer is about one third: that is, the gain rises by 4%, compared to a maximum in principle of 14%.

Other points to note are the very poor performance of Non-Tradable Targets, already discussed theoretically after Proposition 3; and the very modest abatement level in the Reference Case that has effectively been chosen by our calibration of the global benefit parameter V. Abatement $\bar{Q}$ of 8.6 Gt/yr is only 16% of BAU emissions, and abated emissions in 2020 still grow from the 2000 baseline, though by about a third as much as the 32% growth under BAU. This modesty is echoed (for example) in the certainty element of abatement cost, $\bar{C}$ reported at the bottom of the Table, which is 0.1% of projected global GDP in 2020. This reassures us that this is an acceptable application of a partial equilibrium model like MAGES.

3 RESULTS UNDER RISK AVERSION AND ENDOGENOUS TARGETS

In Section 3 we assume that all parties are risk-averse, with their welfare from a shift to a global abatement mechanism being better represented by a strictly concave function of the resulting gain net benefit, rather than the gain itself. Few studies have include risk aversion in analysing global emissions control under uncertainty (Bohm and Carlen 2002 being a notable exception), and in our view such inclusion is a vital step towards politically more realistically analyses. In Section 3.1 we compute expected global payoff under emissions trading, and in Section 3.2 we do the same for a tax with thresholds. Non-tradable targets are omitted, for reasons explained below in Section 3.3, which focuses on the way total abatement changes when it is global payoff rather than gain which is maximised, and on the choice of equity criterion which is then needed to make payoff maximisation result in a sensible distribution of abatement targets among parties. Section 3.4 gives the empirical results for expected global payoff and abatement for the case of GHG control in 2020.

To allow non-linear expectations to be computed, here we assume that all errors are normal (and still independent with mean zero)

$$\varepsilon_{Yi} \sim N(0,\sigma_{Yi}),\quad \varepsilon_{\eta i} \sim N(0,\sigma_{\eta i}),\quad \varepsilon_{\rho i} \sim N(0,\sigma_{\rho i}),\quad \varepsilon_{Ci} \sim N(0,\sigma_{Ci}) \qquad [3.1]$$

So defining $\varepsilon := (\varepsilon_{Y1},\ldots,\varepsilon_{Yn},\varepsilon_{\eta 1},\ldots,\varepsilon_{\eta n},\varepsilon_{\rho 1},\ldots,\varepsilon_{\rho n},\varepsilon_{C1},\ldots,\varepsilon_{Cn})$, the expectation of any random variable $\tilde{J}$ is

$$E[\tilde{J}] := \int_{\mathbb{R}^{4n}} \tilde{J}(\varepsilon)\, \Big[e^{-\frac{1}{2}\Sigma[(\varepsilon_{Yi}/\sigma_{Yi})^2+(\varepsilon_{\eta i}/\sigma_{\eta i})^2+(\varepsilon_{\rho i}/\sigma_{\rho i})^2+(\varepsilon_{Ci}/\sigma_{Ci})^2]} \Big/ (2\pi)^{2n}\Pi_{i=1}^{n}(\sigma_{Yi}\sigma_{\eta i}\sigma_{\rho i}\sigma_{Ci})\Big]\, d\varepsilon \qquad [3.2]$$

3.1 Expected payoffs from Emissions Trading

Appendix 1 shows that party i's realised net benefit from ET is

$$\tilde{A}^T{}_i = \bar{A}^T{}_i + \tilde{F}^T{}_i - p^T(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + H^T{}_{Ei}\tilde{N}_E + H^T{}_{Ci}\tilde{N}_C \qquad [3.3]$$

where

$$\bar{A}^T{}_i(\mathbf{x}) := V_i p^T M + ½(p^T)^2(M_i - W_i M^2) - p^T(E^b{}_i - X^T{}_i) \quad \$/\text{yr}, \qquad [3.4]$$

$$\tilde{F}^T{}_i(\mathbf{x}) := ½(M_i/M^2 - W_i)\tilde{N}_E{}^2 - (1/M)\tilde{N}_{Ei}{}^2 + ½(M_i/M^2)\tilde{N}_C{}^2$$
$$+ [½(1/M_i) - 1/M]\tilde{N}_{Ci}{}^2 + (M_i/M^2)\tilde{N}_E\tilde{N}_C$$
$$- (1/M)[\tilde{N}_{Ci}(\tilde{N}_{C-i}+\tilde{N}_E)+\tilde{N}_{Ei}(\tilde{N}_{E-i}+\tilde{N}_C)] \quad \$/\text{yr} \qquad [3.6]$$

$$H^T{}_{Ei}(\mathbf{x}) := (p^T M_i - E^b{}_i + X^T{}_i)/M + V_i - W_i p^T M \quad \$/\text{t, and} \qquad [3.7]$$

$$H^T{}_{Ci}(\mathbf{x}) := (p^T M_i - E^b{}_i + X^T{}_i)/M \quad \$/\text{t}. \qquad [3.8]$$

For any mechanism j, we define the term $\tilde{F}^j{}_i$ to contain all the squared and cross-multiplied errors (terms in $\varepsilon_{Yi}{}^2$ and $\varepsilon_{Yi}\varepsilon_{Ck}$, etc) in $\tilde{A}_i$, but no other errors. This makes the expectation of all the other error terms (here the last three terms of [3.3]) zero. We also define $\bar{A}^j{}_i$ as what would be the expectation of $\tilde{A}^j{}_i$ under certainty. For ET the expected net benefit is then

$$A^T{}_i(\mathbf{x}) = \bar{A}^T{}_i + F^T{}_i, \quad \text{where} \qquad [3.9]$$

$$F^T{}_i(\mathbf{x}) = -(1/M)D_{Ei} - ½(W_i - M_i/M^2)D_E$$
$$+ [½(1/M_i) - 1/M]D_{Ci} + ½(M_i/M^2)D_C. \qquad [3.11]$$

It is also worth for future reference giving the variants of [3.11] for the special cases of absolute and intensity targets, respectively:

$$F^{T0}{}_i := ½(M_i/M^2 - W_i)D^0{}_E - (1/M)D^0{}_{Ei} + ½(M_i/M^2)D_C + [½(1/M_i) - 1/M]D_{Ci}, \qquad [3.12]$$

$$F^{T*}{}_i := ½(M_i/M^2 - W_i)D^*{}_E - (1/M)D^*{}_{Ei} + ½(M_i/M^2)D_C + [½(1/M_i) - 1/M]D_{Ci}. \qquad [3.13]$$

From [2.35], [2.36] and [2.77a], *i*'s *payoff* from moving from unilateral abatement to ET is:

$$\tilde{U}^T_i \;:= \tilde{A}^T_i - A^U_i + (z/y_i)(1-e^{-r(\tilde{A}^T_i - A^U_i)}). \qquad [3.16]$$

Expected payoff from this move is then

$$U^T_i \;= E[\tilde{U}^T_i] = \bar{A}^T_i + F^T_i - A^U_i + z/y_i - (z/y_i)E[e^{-r(\tilde{A}^T_i - A^U_i)}] \qquad [3.17]$$

For j = either T (ET) or \$ (the tax case, dealt with in the next section), we now define

$\mathbf{Z}$:= the $4n$ x $4n$ dispersion matrix of our global model, so given our assumption that all $4n$ errors are independent, the inverse dispersion matrix $\mathbf{Z}^{-1}$ is diagonal, with $1/\sigma_{Y1}^2$, $1/\sigma_{\eta 1}^2$, $1/\sigma_{\rho 1}^2$, $1/\sigma_{C1}^2,\ldots,1/\sigma_{Yn}^2$, $1/\sigma_{\eta n}^2$, $1/\sigma_{\rho n}^2$, $1/\sigma_{Cn}^2$ on the diagonal and zeros elsewhere; [3.17a]

$\mathbf{F}^j_i$:= a $4n\times 4n$, non-unique matrix such that $\varepsilon'\mathbf{F}^j_i\varepsilon \equiv \tilde{F}^j_i$ in [3.6] (or [3.29] below for the tax case); [3.17b]

$\mathbf{S}^j_i := \mathbf{Z}^{-1}+2r\mathbf{F}^j_i$; [3.17c]

$\mathbf{h}^j_i$ is a $4n$-vector that generates the 1st order errors in $\tilde{A}^j_i$, i.e. the last three terms in [3.3] or [3.27] (not just the H-terms), formally defined by $(\mathbf{h}^j_i)'\varepsilon \equiv \tilde{A}^j_i - \bar{A}^j_i - \tilde{F}^j_i$; [3.17d]

′ denotes transpose;

$\mathbf{F}^{j0}_i$, $\mathbf{S}^{j0}_i$ and $\mathbf{h}^{j0}_i$ for absolute targets, and $\mathbf{F}^{j*}_i$, $\mathbf{S}^{j*}_i$ and $\mathbf{h}^{j*}_i$ for optimal intensity targets, are defined in the obvious way using [2.22] and [2.24] respectively. [3.17e]

It is then shown in Appendix 5 that

$$E[\exp\{-r(\tilde{A}^j_i - A^U_i)\}] \;= (|\mathbf{S}^j_i|/|\mathbf{Z}|)^{½} \exp\{½r^2(\mathbf{h}^j_i)'(\mathbf{S}^j_i)^{-1}\mathbf{h}^j_i - r(\bar{A}^j_i - A^U_i)\}, \qquad [3.17f]$$

so that our generic final result is

$$U^j_i \quad = \bar{A}^j_i + F^j_i - A^U_i + z/y_i$$
$$- (z/y_i)(|\mathbf{S}^j_i|/|\mathbf{Z}|)^{½} \exp\{½r^2(\mathbf{h}^j_i)'(\mathbf{S}^j_i)^{-1}\mathbf{h}^j_i - r(\bar{A}^j_i - A^U_i)\}. \qquad [3.18]$$

Applying this to the ET case and using [2.22] and [2.24] then gives risk-averse payoffs for ET with absolute and intensity targets, for which we report our empirical results:

Proposition 4: Party i's expected payoff from ET with:

(a) absolute targets compared to unilateralism is

$$U^{T0}_i \quad = \bar{A}^T_i + F^{T0}_i - A^U_i + z/y_i$$
$$- (z/y_i)(|\mathbf{S}^{T0}_i|/|\mathbf{Z}|)^{½} \exp\{½r^2(\mathbf{h}^{T0}_i)'(\mathbf{S}^{T0}_i)^{-1}\mathbf{h}^{T0}_i - r(\bar{A}^T_i - A^U_i)\} [3.22]$$

(b) (optimal) intensity targets compared to unilateralism is

$$U^{T*}_i \quad = \bar{A}^T_i + F^{T*}_i - A^U_i + z/y_i$$
$$- (z/y_i)(|\mathbf{S}^{T*}_i|/|\mathbf{Z}|)^{½} \exp\{½r^2(\mathbf{h}^{T*}_i)'(\mathbf{S}^{T*}_i)^{-1}\mathbf{h}^{T*}_i - r(\bar{A}^T_i - A^U_i)\} [3.23]$$

where $\bar{A}^T_i$ *is as in [3.4],* F^{T0}_i *and* F^{T*}_i *are as in [3.12]-[3.13],* A^U_i *is as in [2.74], and the* **S**, **Z** *and* **h** *matrices are as in [3.17a]-[3.17e].*

For detailed empirical results by country and globally, see JP.

3.2 Expected payoffs for a tax with thresholds

Appendix 2 shows that

$$\tilde{A}^\$_i = \bar{A}^\$_i + \tilde{F}^\$_i - p^\$(\tilde{N}_{Ei} + \tilde{N}_{Ci}) + H^\$_{Ei}\tilde{N}_E + H^\$_{Ci}\tilde{N}_C, \qquad [3.27]$$

where

$$\bar{A}^{\$}_i := V_i p^{\$} M - \tfrac{1}{2} W_i (p^{\$})^2 M^2 + \tfrac{1}{2}(p^{\$})^2 M_i - p^{\$}(E^b_i - X^{\$}_i) \qquad [3.28]$$

$$\tilde{F}^{\$}_i := \tfrac{1}{2}(1/M_i)\tilde{N}_{Ci}^{\,2} - \tfrac{1}{2} W_i \tilde{N}_C^{\,2}, \qquad [3.29]$$

$$H^{\$}_{Ei} := (X^{\$}_i / X^{\$}) p^{\$}, \text{ and} \qquad [3.31]$$

$$H^{\$}_{Ci} := (X^{\$}_i / X^{\$}) p^{\$} - V_i + W_i p^{\$} M \qquad [3.32]$$

The expected net benefit is then

$$A^{\$}_i = \bar{A}^{\$}_i + F^{\$}_i, \text{ where } F^{\$}_i = (1/2M_i) D_{Ci} - \tfrac{1}{2} W_i D_C. \qquad [3.34]$$

We can then define $\mathbf{F}^{\$0}_i$, $\mathbf{S}^{\$0}_i$ and $\mathbf{h}^{\$0}_i$ and $\mathbf{F}^{\$*}_i$, $\mathbf{S}^{\$*}_i$ and $\mathbf{h}^{\$*}_i$ using [3.17b]-[3.17e] and [3.27]-[3.32]. Inserting these into the generic final result [3.18] then gives:

Proposition 5: Party i's expected payoff from a tax and

(a) absolute thresholds compared to unilateralism is:

$$U^{\$0}_i = \bar{A}^{\$}_i + F^{\$0}_i - A^U_i + z/y_i$$
$$- (z/y_i)(|\mathbf{S}^{\$0}_i|/|\mathbf{Z}|)^{1/2} \exp\{\tfrac{1}{2} r^2 (\mathbf{h}^{\$0}_i)'(\mathbf{S}^{\$0}_i)^{-1}\mathbf{h}^{\$0}_i - r(\bar{A}^{\$}_i - A^U_i)\}. \qquad [3.43]$$

(b) (optimal) intensity thresholds compared to unilateralism is:

$$U^{\$*}_i = \bar{A}^{\$}_i + F^{\$*}_i - A^U_i + z/y_i$$
$$- (z/y_i)(|\mathbf{S}^{\$*}_i|/|\mathbf{Z}|)^{1/2} \exp\{\tfrac{1}{2} r^2 (\mathbf{h}^{\$*}_i)'(\mathbf{S}^{\$*}_i)^{-1}\mathbf{h}^{\$*}_i - r(\bar{A}^{T}_i - A^U_i)\}. \qquad [3.44]$$

where $\bar{A}^{\$}_i$ is as in [3.28], $F^{\$}_i$ is as in [3.34], and A^U_i is as in [2.74].

3.3 Optimisation, endogenous targets and abatement, and equity criteria

Under Section 2's assumption that parties are risk-neutral, the optimal policy for any mechanism was to maximise global expected net benefit A. In summing the party-level results A_i to produce A, the dependence of A_i on the target distribution $\{x_i\}$ cancelled out as long as absolute or intensity targets were used, and dependence of the $\{D_{Ei}\}$ on $\{x_i\}$ was thus avoided. Optimisation then yielded the simplified results in Proposition 3.

In Section 3, with risk-averse parties, the optimal policy is to maximise global expected payoff U, and the A_i occur in the exponential terms as well as in the additive terms of U_i; so they do not just sum to A when the U_i are summed. In particular, the dependences of all $\bar{A}_i$ on x_i in [3.4] remain in U, even if absolute or (optimal) intensity targets are used so that all D_{Ei} do not depend on x_i. In turn this means that the optimal choice of target total X (hence price $p(X)$), and its distribution $\{X_i\}$, depend on how well the mechanism chosen neutralises uncertainty costs: that is, targets (hence total abatement and the emission price) are *endogenous to mechanism choice*, unlike in Proposition 3. This is an important step, since making targets endogenous will increase the difference in payoff achieved by moving to a mechanism that neutralises more uncertainty, compared to the difference calculated with exogenous targets, that is, targets fixed by some rule that does not allow them to vary across mechanisms. It also explains why NTTs are excluded from the risk-averse analysis. Optimising expected global payoff with NTTs would require each party's expected shadow price to be uniquely endogenous to that party, which then invalidates the assumption of a common shadow price needed to reach results like [2.62].

With endogenous targets, unconstrained maximisation of U will determine an 'optimal' target distribution. However, for our empirical GHG case, this distribution turns out to be politically nonsensical, with many parties having zero targets. Since our primary motive for introducing risk aversion was to increase the political realism of the results, we must therefore constrain target distribution to meet some equity criterion. Maximising payoff U subject to this equity constraint, together with a chosen standard mechanism type, then comprise a reference case which we can use as a baseline when comparing the effectiveness of different mechanisms.

In our GHG empirical example, the *equity criterion* we use is that:

Targets $\{x_i\}$ are distributed so that each country's optimised, expected payoff per person is equal: $U_i/L_i = U_k/L_k$ for all i, k; and [3.46]

the *Reference Case* now comprises the use of *emissions trading with absolute targets* as an abatement mechanism, and the maximisation of expected global payoff (U^{T0}) subject to criterion [3.46]. [3.47]

JP discusses alternative criteria that could have been used, and how sensitive the optimal payoff is to the choice of criterion in the GHG case. The basic finding is that although the optimal distribution $\{x_i\}$ obviously varies a lot across criteria, the comparison between absolute and intensity targets for ET in terms of expected global payoff is little affected.

Empirically, target distributions that simultaneously fulfil the optimality and equity criteria are computed in a two-stage iterative algorithm. In a first set of model runs, for a given overall target X, country targets X_i are varied until equal expected per capita payoff U (as in [3.46]) is achieved, to within 0.5% for each country i. This is then repeated for different aggregate target levels X and thus levels of expected abatement Q, until the scenario with globally optimal expected payoff U (and equal per capita payoff u_i) is found.

3.4 Empirical, risk-averse results for GHGs

Expected global results for risk-averse optimisation of GHG abatement in 2020 are reported in **Table 3**, which is laid out like Table 2 to make comparisons easier. The empirical calibration for the GHG case is as in Section 2.7, except for additional calibration of the risk aversion parameters

Table 3 Results for GHG abatement mechanisms under risk-aversion (equity-constrained maximisation of expected global payoff)

% figures are changes from the **Reference Case** (Emissions Trading with Absolute targets)

Mechanism	Expected global payoff vs. unilateralism (G$/yr)		Expected global abatement (Gt/yr)	
	Type of thresholds or targets			
	Absolute	Optimal Intensity	Absolute	Optimal Intensity
Certainty case: any mechanism	$\bar{U}(\bar{Q})$ 76.7 (61%)		$\bar{Q}$ 8.60 (+32%)	
Tax with Thresholds	$U^{\$0}(Q^{\$0})$ 74.3 (+56%)	$U^{\$*}(Q^{\$*})$ 75.5 (+59%)	$Q^{\$0}$ 8.10 (+25%)	$Q^{\$*}$ 8.34 (+28%)
Emissions Trading	$\boldsymbol{U^{T0}(Q^{T0})}$ **47.6**	$U^{T*}(Q^{T*})$ 60.8 (+28%)	$\boldsymbol{Q^{T0}}$ **6.51**	Q^{T*} 7.15 (+10%)
Non-Tradable Targets	n/a			
Unilateral case has abatement $\bar{Q}^U \approx 0.90$ Gt/yr, and expected net benefits $\bar{A}^U(\bar{Q}^U) \approx 18.2$ G$/yr under certainty, and $A^U(\bar{Q}^U) \approx 24.5$ G$/yr under uncertainty.				

$\{z_i\}$ and r. As with $\{V_i\}$ and $\{W_i\}$, this is done by inference, this time to a produce plausible difference in global abatement between a 'reference case', which uses ET with absolute targets, and the same case with risk neutrality ($r = 0$). We also of course switch the welfare measure being maximised and reported from expected global gain in net benefit compared to unilateralism, $G = A - A^U$, to expected global payoff versus unilateralism, $U(\{G_i(Q)\})$. So the first two columns of numbers in Table 3 give payoffs U in place of the gains G reported in Table 2.

The ranking of the mechanisms here remains unchanged from the risk-netural ranking in Table 2. Price-based control (a tax) is preferable to quantity-based control (emissions trading), and within that, (optimal) indexation is preferable to no indexation (absolute targets/thresholds). Endogeneity of abatement under risk aversion means that optimal global abatement now varies with the mechanism chosen, so we can now assess changes in environmental effectiveness of the treaty, in addition to changes in welfare. In turn this variation is shown in the arguments of expected payoffs, and in particular this divides the tax case into separate results for optimal intensity and absolute thresholds.

Under our calibration, taxes with optimal intensity thresholds (the best mechanism) yield a 28% increase in global abatement compared to our reference case of ET with absolute targets, thus bridging seven eighths of the gap to the certainty case; while non-indexed taxes increase expected abatement by 25%. ET with optimal indexation gives a 16% increase in expected global abatement over its non-indexed equivalent. Politically, activity-indexed tax thresholds might be much less realistic than activity-indexed targets, so the relevant comparison between prices and quantities may be between the 25% increase under non-indexed taxes and the 10% increase under indexed targets.

Expected payoffs show the same ranking as expected abatement, but with very different magnitudes of differences between mechanisms. All uncertain payoffs are significantly lower than the corresponding gains in

Table 2, thanks primarily to the final, risk-aversion terms starting in z_i (here $1/y_i$, as in Table 1) that are found in Proposition 4 and Proposition 5. But because risk aversion is included, the drop is greatest ($G^{T0}(\bar{Q})-U^{T0}(Q^{T0})$) for the Reference Case, which is the mechanism that is already the worst available one for neutralising risk; and least ($G^{\$*}(\bar{Q})-U^{\$*}(Q^{\$*})$) for a tax with optimal intensity thresholds, the best available mechanism. Whereas switching from the former to the latter improved risk-neutral gain by 14% in Table 2, here it improves risk-averse payoff by about 4 times as much. So the welfare differences between ET and a tax, and between indexed and non-indexed ET, are amplified under risk aversion. However, in contrast to the abatement results, there is only a small gap between expected payoff under indexed and non-indexed taxes – this is because expected payoff as a function of expected abatement is relatively flat around the optimum, in the tax cases.

Not shown in the Table but worth reporting briefly is the wide range found in JP (Table III) of optimal indexation, $\beta_i = \alpha_i/x_i$, across countries. The high is $\beta_i = 1.30$ for Japan, with most ($\alpha_i = 0.96$) of emissions linked to GDP and a stringent ($x_i = 0.739$) target; and the low is 0.21 for Indonesia, with the least (0.19) linkage, and a more generous (0.914) target.

4. CONCLUSIONS

This paper has introduced a model of Mechanisms to Abate Global Emissions under Stochasticity (MAGES), which gives both risk-neutral and risk-averse analyses of mechanisms for abating well-mixed ('global'), future (hence uncertain) emissions from many parties. The model is partial equilibrium in nature, and so is appropriate to circumstances where abatement costs are small relative to parties' GDPs (if countries) or revenues (if firms). It generalises Weitzman's classic (1974) stochastic comparison of 'price' vs. 'quantity' mechanisms in four main ways at the same time.

First, it applies to many parties, and thus incorporates tradable targets ('emissions trading' or ET) as well as a tax, and non-tradable targets (NTTs), with all three mechanisms designed to be globally revenue-neutral. For a tax this entails not just the use of thresholds equivalent to grandfathered target allocations to make it revenue-neutral on average, but also a pro-rata, lump-sum rebate of net global tax revenue to make it revenue-neutral in every realisation of the future.

Second, it includes uncertainties in future business-as-usual emissions − in fact three uncertainties, only one of which is linked to a party's activity level − as well as in marginal abatement costs (MACs). Third, all 'quantities', whether target allocations, or thresholds for a tax, are flexible, being continuously indexed to parties' activity levels, which includes the concept of intensity targets, though here with indexation being optimally tailored to each party.

Fourth, MAGES allows us to analyse optimal policies under both risk-neutral behaviour, where welfare is defined as expected global net benefit, and risk-averse behaviour, where welfare (dubbed 'payoff') is a strictly concave function of net benefit. The latter assumption requires the use of more speculative data, but we feel it is likely to yield a more realistic indicator of parties' reactions to policy choices. Crucially, it also allows us to model how a better abatement mechanism can achieve not just higher welfare, but also a better environmental outcome.

Under risk neutrality, several insights followed immediately from the MAGES results for expected global net benefit under the three mechanisms. First, the tax neutralises all emissions uncertainties (i.e. prevents them from affecting net benefit), NTTs neutralises all MAC uncertainties, and ET neutralises neither, differences which reflect the mechanisms' different basic modes of operation. Second, indexation can neutralise some of the uncertainty costs inherent in emission targets. Third, under ET, higher marginal abatement cost uncertainty is an unambiguous benefit(!) compared to certainty. Fourth, because the steepness of the marginal benefit curve decreases net benefit under both ET (thanks to emissions uncertainty) and a tax (thanks to MAC uncertainty, as in Weitzman), there is no longer a simple rule for preferring 'prices' (a tax) to 'quantities' (ET). Fifth, the gains from making targets tradable are simple to compute, and large.

Under risk aversion, the endogeneity of the expected global abatement level prevents simple theoretical comparisons among mechanisms, but we still found near-analytic formulae for expected global payoff in terms of net global emissions uncertainties.

Empirically, we applied both risk-neutral and risk-averse versions of the MAGES theory to greenhouse gas (GHG) abatement in 2020, using MAGES-GHG, an 18-region numerical simulation model with new estimates for BAU emissions uncertainties. Here a party is a country, its activity level is GDP, and only part of its emissions are linked to GDP. Among the key global results are that under risk neutrality, a tax raises welfare by 14% above ET, which itself is vastly better than non-tradable targets. The tax's dominance is well-known in the context of climate policy. However, we hope that its presentation here, as an automatically revenue-neutral mechanism, may lift some interest in it away from the economist's understandable but forlorn interest in revenue-raising mechanisms, towards a politically more realistic revival of the carbon tax option.

Under risk aversion, uncertainty lowers welfare by much more under ET than under a tax; while NTTs are no longer computable, because a single shadow price is no longer optimal. Nevertheless we use the worst feasible mechanism − ET with unindexed targets − as our reference case, because it is politically most accepted. Compared to this, an optimally indexed tax gives about 60% more welfare and 30% more abatement, while optimally indexed ET achieves roughly two-fifths of these maximal improvements.

The broad picture we paint is therefore that taxes remain the best instrument under uncertainty with flat marginal benefits, but they need thresholds to be politically acceptable, and ideally, those thresholds should also be flexible. So far, this message has not been received in the world of policymaking, where quantity instruments (emissions trading schemes) are on the rise; but these should at least have flexible targets, as long as the extra complexity arising remains manageable.

Our results suggest several different strands of further work. The first and foremost is to measure the scope for free riding, by deriving results for parties joining or leaving an abatement policy one by one, rather than altogether. One is to apply them to other pollutants which are genuinely 'global' in nature, or have been treated as such by current policies (the obvious example being SO_2 trading in the USA). Another is to compare the results here with those of Pizer (2002), Jacoby and Ellerman (2004) and others working on hybrids between price and quantity control, such as ET with a price cap (a 'safety valve'). Any price cap is effectively a truncation condition which will complicate taking multinormal expectations, but may be possible using techniques we have developed in preliminary work on applying MAGES to the idea of non-binding targets (following up an idea in Philibert 2000). And our rebate mechanism that is essential to achieving global revenue-neutrality for a tax with thresholds raises many practical and policy questions that deserve further exploration.

APPENDIX 1 NET BENEFIT FROM EMISSIONS TRADING

From [2.48] with $j = T$, party i's realised net benefit under ET is

$$\tilde{A}^T_i := V_i\tilde{Q}^T - ½W_i(\tilde{Q}^T)^2 + \tilde{R}^T_i - ½(1/M_i)\tilde{Q}^{T\,2}_i - (1/M_i)\tilde{Q}^T_i\tilde{N}_{Ci} \qquad [A1.1]$$

Next, from [3.3], permit sales revenue is

$$\tilde{R}^T_i := \tilde{p}^T(\tilde{X}^T_i - \tilde{E}^T_i), \text{ which by [2.3] and [2.14]}$$

$$= \tilde{p}^T[\tilde{Q}^T_i - (E^b_i - X^T_i + \tilde{N}_{Ei})], \text{ which by [2.49]}$$

$$= \tilde{p}^T(\tilde{p}^T M_i - \tilde{N}_{Ci} - E^b_i + X^T_i - \tilde{N}_{Ei}). \qquad [A1.2]$$

Combining [A1.1] with [2.52], [A1.2] and [2.49] then gives

$$\tilde{A}^T_i = V_i(p^T M + \tilde{N}_E) - ½W_i(p^T M + \tilde{N}_E)^2 + \tilde{p}^T(\tilde{p}^T M_i - \tilde{N}_{Ci} - E^b_i + X^T_i - \tilde{N}_{Ei})$$

$$- ½(1/M_i)(\tilde{p}^T M_i - \tilde{N}_{Ci})^2 - (1/M_i)(\tilde{p}^T M_i - \tilde{N}_{Ci})\tilde{N}_{Ci} \qquad [A1.3]$$

which can be shown by lengthy algebra *[see Referees' Appendix 2]* to be

$$\tilde{A}^T_i = \bar{A}^T_i + \tilde{F}^T_i - p^T(\tilde{N}_{Ei} + \tilde{N}_{Ci}) + H^T_{Ei}\tilde{N}_E + H^T_{Ci}\tilde{N}_C \qquad \text{as in [3.3]}$$

where

$$\bar{A}^T_i := V_i p^T M + ½(p^T)^2(M_i - W_i M^2) - p^T(1-x_i)E^b_i \ \text{\$/yr}, \qquad \text{as in [3.4]}$$

$$\tilde{F}^T_i := ½(M_i/M^2 - W_i)\tilde{N}_E^{\,2} - (1/M)\tilde{N}_{Ei}^{\,2} + ½(M_i/M^2)\tilde{N}_C^{\,2}$$

$$+ [½(1/M_i) - 1/M]\tilde{N}_{Ci}^{\,2} + (M_i/M^2)\tilde{N}_E\tilde{N}_C$$

$$- (1/M)[\tilde{N}_{Ci}(\tilde{N}_{C-i} + \tilde{N}_E) + \tilde{N}_{Ei}(\tilde{N}_{E-i} + \tilde{N}_C)] \qquad \text{as in [3.6]}$$

$$H^T_{Ei} := (p^T M_i - E^b_i + X^T_i)/M + V_i - W_i p^T M, \text{ and} \qquad \text{as in [3.7]}$$

$$H^T_{Ci} := (p^T M_i - E^b_i + X^T_i)/M, \qquad \text{as in [3.8].}$$

Summing all these results gives global realised net benefit:

$$\tilde{A}^T = \bar{A}^T + \tilde{F}^T - p^T(\tilde{N}_E + \tilde{N}_C) + H^T_E\tilde{N}_E, \text{ where} \qquad [A1.4]$$

$$\bar{A}^T = p^T VM + ½(p^T)^2(M - WM^2) - p^T Q^T, \text{ which since } Q^T = p^T M$$

$$= p^T VM - ½(p^T)^2 M(1 + WM); \qquad \text{as in [2.63]}$$

$$\tilde{F}^T = ½(1/M-W)\tilde{N}_E{}^2 - (1/M)\Sigma\tilde{N}_{Ei}{}^2 + ½(1/M)\tilde{N}_C{}^2$$
$$+ ½\Sigma(1/M_i)\tilde{N}_{Ci}{}^2 - (1/M)\Sigma\tilde{N}_{Ci}{}^2 + (1/M)\tilde{N}_E\tilde{N}_C$$
$$- (1/M)[\Sigma\tilde{N}_{Ci}(\tilde{N}_{C-i}+\tilde{N}_E)+\Sigma\tilde{N}_{Ei}(\tilde{N}_{E-i}+\tilde{N}_C)] \quad \text{[A1.7]}$$
$$H^T{}_E = V - Wp^TM, \;\; H^T{}_C = 0. \quad \text{[A1.8]}$$

Finally, taking expectations gives

$A^T = \bar{A}^T + F^T$, where $\bar{A}^T$ is as in [2.63], and [A1.9]

$$F^T := ½(1/M-W)D_E - (1/M)D_E + ½(1/M)\Sigma D_{Ci} + ½\Sigma(1/M_i)D_{Ci} - (1/M)D_C$$
$$= - ½\Sigma(1/M+W)D_{Ei} + ½\Sigma(1/M_i-1/M)D_{Ci}. \quad \text{as in [2.60]}$$

The variants with absolute or intensity targets are thus

$$A^{T0} = \bar{A}^T - ½\Sigma(1/M+W)D^0{}_{Ei} + ½\Sigma(1/M_i-1/M)D_{Ci}, \quad \text{as in [2.65],}$$
& $$A^{T*} = \bar{A}^T - ½\Sigma(1/M+W)D^*{}_{Ei} + ½\Sigma(1/M_i-1/M)D_{Ci}. \quad \text{as in [2.66]}$$

As before, $\bar{A}^T$ contains no error variances, so does not need the ^{0} superscript.

APPENDIX 2 NET BENEFIT FROM TAX WITH THRESHOLDS

From [2.48] and [2.32], party i's realised net benefit under a tax with thresholds, compared to no abatement anywhere, is

$$\tilde{A}^\$_i := V_i\tilde{Q}^\$ - ½W_i(\tilde{Q}^\$)^2 + \tilde{K}^\$_i - (X^\$_i/X^\$)\tilde{K}^\$ - ½(1/M_i)(\tilde{Q}^\$_i)^2$$
$$- (1/M_i)\tilde{Q}^\$_i\tilde{N}_{Ci} \quad \text{[A2.1]}$$

From [2.29], and then [2.54] and [2.14],

$$\tilde{K}^\$_i = p^\$[\tilde{Q}^\$_i-(\tilde{E}^b{}_i-\tilde{X}^\$_i)]$$
$$= p^\$(p^\$M_i+X^\$_i-E^b{}_i-\tilde{N}_{Ei}-\tilde{N}_{Ci}), \;\; \text{hence} \quad \text{[A2.2]}$$
$$\Rightarrow \tilde{K}^\$ = -p^\$(\tilde{N}_E+\tilde{N}_C). \quad \text{[A2.3]}$$

Inserting these and [2.54] into [A2.1] then gives, after some algebra:[8]

$$\tilde{A}^{\$}_i = \bar{A}^{\$}_i + \tilde{F}^{\$}_i - p^{\$}(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + H^{\$}_{Ei}\tilde{N}_E + H^{\$}_{Ci}\tilde{N}_C \qquad \text{as in [3.27],}$$

where

$$\bar{A}^{\$}_i := V_i p^{\$} M - ½W_i(p^{\$})^2 M^2 + ½(p^{\$})^2 M_i - p^{\$}(E^b{}_i - X^{\$}{}_i) \qquad \text{as in [3.28]}$$

$$\tilde{F}^{\$}_i := (1/2M_i)\tilde{N}_{Ci}{}^2 - ½W_i\tilde{N}_C{}^2, \qquad \text{as in [3.29]}$$

$$H^{\$}_{Ei} := (X^{\$}{}_i/X^{\$})p^{\$}, \qquad \text{as in [3.31], and}$$

$$H^{\$}_{Ci} := (X^{\$}{}_i/X^{\$})p^{\$} - V_i + W_i p^{\$} M \qquad \text{as in [3.32].}$$

The variant with absolute rather than flexible targets comes from adding a ^{0} superscript to terms with D_E variables, but there are none such here, so

$$F^{\$0}{}_i = F^{\$}{}_i \text{ and } A^{\$0}{}_i = A^{\$}{}_i. \qquad \text{[A2.4]}$$

The corresponding global sums are:

$$\tilde{A}^{\$} = \bar{A}^{\$} + \tilde{F}^{\$} - p^{\$}\tilde{N}_C + H^{\$}{}_C\tilde{N}_C, \text{ where} \qquad \text{[A2.6]}$$

$$\bar{A}^{\$} := p^{\$}VM - ½(p^{\$})^2 M(1+WM); \qquad \text{[A2.7]}$$

$$\tilde{F}^{\$} := ½\Sigma(1/M_i)\tilde{N}_{Ci}{}^2 - ½W\tilde{N}_C{}^2; \text{ and} \qquad \text{[A2.8]}$$

$$H^{\$}{}_C := p^{\$}(1+WM) - V. \qquad \text{[A2.9]}$$

Expected global net benefit is then

$$A^{\$} = \bar{A}^{\$} + ½\Sigma(1/M_i - W)D_{Ci} \qquad \text{as in [2.61].}$$

---

8. The intermediate working is

$$\begin{aligned}
\tilde{A}^{\$}{}_i := {} & V_i(p^{\$}M - \tilde{N}_C) - ½W_i(p^{\$}M - \tilde{N}_C)^2 + \mathbf{[}p^{\$}(p^{\$}M_i + X^{\$}{}_i - E^b{}_i - \tilde{N}_{Ei} - \tilde{N}_{Ci}) \\
& + (X^{\$}{}_i/X^{\$})p^{\$}(\tilde{N}_E + \tilde{N}_C) - ½(1/M_i)(p^{\$}M_i - \tilde{N}_{Ci})^2 - (1/M_i)(p^{\$}M_i - \tilde{N}_{Ci})\tilde{N}_{Ci}\mathbf{]} \\
= {} & V_i p^{\$}M - ½W_i(p^{\$})^2M^2 - (V_i - W_i p^{\$}M)\tilde{N}_C - ½W_i\tilde{N}_C{}^2 \\
& + \mathbf{[}(p^{\$})^2M_i - p^{\$}(E^b{}_i - X^{\$}{}_i) - p^{\$}\tilde{N}_{Ei} - p^{\$}\tilde{N}_{Ci} + (X^{\$}{}_i/X^{\$})p^{\$}(\tilde{N}_E + \tilde{N}_C) \\
& - ½(p^{\$})^2M_i + p^{\$}\tilde{N}_{Ci} - (1/2M_i)\tilde{N}_{Ci}{}^2 - p^{\$}\tilde{N}_{Ci} + (1/M_i)\tilde{N}_{Ci}{}^2\mathbf{]} \\
= {} & V_i p^{\$}M - ½W_i(p^{\$})^2M^2 + ½(p^{\$})^2M_i - p^{\$}(E^b{}_i - X^{\$}{}_i) - (V_i - W_i p^{\$}M)\tilde{N}_C \\
& - p^{\$}(\tilde{N}_{Ei} + \tilde{N}_{Ci}) + (X^{\$}{}_i/X^{\$})p^{\$}(\tilde{N}_E + \tilde{N}_C) - ½W_i\tilde{N}_C{}^2 + ½(1/M_i)\tilde{N}_{Ci}{}^2 \\
= {} & V_i p^{\$}M - ½W_i(p^{\$})^2M^2 + ½(p^{\$})^2M_i - p^{\$}(E^b{}_i - X^{\$}{}_i) + ½(1/M_i)\tilde{N}_{Ci}{}^2 - ½W_i\tilde{N}_C{}^2 \\
& - p^{\$}(\tilde{N}_{Ei} + \tilde{N}_{Ci}) + (X^{\$}{}_i/X^{\$})p^{\$}\tilde{N}_E + [-V_i + W_i p^{\$}M + (X^{\$}{}_i/X^{\$})p^{\$}]\tilde{N}_C
\end{aligned}$$

APPENDIX 3 NET BENEFIT FROM NON-TRADABLE TARGETS

From [2.48] with $^j = {}^@$ and $\tilde{R}^@{}_i = 0$, party i's realised net benefit is

$$\tilde{A}^@{}_i = V_i\tilde{Q}^@ - ½W_i(\tilde{Q}^@)^2 - ½(1/M_i)(\tilde{Q}^@{}_i)^2 - (1/M_i)\tilde{Q}^@{}_i\tilde{N}_{Ci}, \qquad [A3.1]$$

which from [2.59] and its sum, $\tilde{Q}^@ = p^@M+\tilde{N}_E$, is

$$= V_i(p^@M+\tilde{N}_E) - ½W_i(p^@M+\tilde{N}_E)^2 - ½(1/M_i)(p^@M_i+\tilde{N}_{Ei})^2$$
$$- (1/M_i)(p^@M_i+\tilde{N}_{Ei})\tilde{N}_{Ci}$$
$$= V_ip^@M - ½W_i(p^@)^2M^2 - ½M_i(p^@)^2 + (V_i - W_ip^@M)\tilde{N}_E - p^@(\tilde{N}_{Ei}+\tilde{N}_{Ci})$$
$$- ½W_i\tilde{N}_E{}^2 - ½(1/M_i)\tilde{N}_{Ei}{}^2 - ½(1/M_i)\tilde{N}_{Ei}\tilde{N}_{Ci}, \text{ so}$$

$$\tilde{A}^@{}_i = \bar{A}^@{}_i + \tilde{F}^@{}_i - p^@(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + H^@{}_{Ei}\tilde{N}_E, \text{ where} \qquad [A3.2]$$

$$\bar{A}^@{}_i := V_ip^@M - ½(M_i+W_iM^2)(p^@)^2, \qquad [A3.3]$$

$$\tilde{F}^@{}_i := -½W_i\tilde{N}_E{}^2 - ½(1/M_i)\tilde{N}_{Ei}{}^2 - ½(1/M_i)\tilde{N}_{Ei}\tilde{N}_{Ci}, \text{ and} \qquad [A3.4]$$

$$H^@{}_{Ei} := V_i - W_ip^@M. \qquad [A3.6]$$

The global totals are then

$$\tilde{A}^@ = \bar{A}^@ + \tilde{F}^@ - p^@(\tilde{N}_E+\tilde{N}_C) + H^@{}_E\tilde{N}_E, \text{ where} \qquad [A3.7]$$

$$\bar{A}^@ := p^@VM - ½M(1+WM)(p^@)^2, \qquad \text{as in [2.62], and}$$

$$\tilde{F}^@ := -½W\tilde{N}_E{}^2 - ½\Sigma(1/M_i)\tilde{N}_{Ei}{}^2 - ½\Sigma(1/M_i)\tilde{N}_{Ei}\tilde{N}_{Ci}, \qquad [A3.8]$$

with expectation

$$A^@ = \bar{A}^@ - ½\Sigma(1/M_i+W)D_{Ei}. \qquad \text{as in [2.62].}$$

The variant with absolute or intensity rather than general flexible targets is obtained by just adding a ^{0} or * superscript to terms containing D_{Ei}, thus

$$A^{@0} = \bar{A}^@ - ½\Sigma(1/M_i+W)D_E{}^0{}_i, \text{ and} \qquad \text{as in [2.68]}$$

$$A^{@*} = ½\bar{p}VM - ½\Sigma(1/M_i+W)D^*{}_{Ei}, \qquad \text{as in [2.69].}$$

Since $\bar{A}^@$ contains no error variances, it does not need the ^{0} superscript.

APPENDIX 4 NET BENEFIT FROM UNILATERALISM

To compute the unilateral case, party i chooses $\tilde{Q}^U{}_i$ to maximise[9]

$$\tilde{A}^U{}_i := V_i\tilde{Q}^U - ½W_i(\tilde{Q}^U)^2 - ½(1/M_i)(\tilde{Q}^U{}_i)^2 - (1/M_i)\tilde{Q}^U{}_i\tilde{N}_{Ci}, \quad \text{as in [2.70]},$$

on the assumption that $\partial\tilde{Q}^U/\partial\tilde{Q}^U{}_i = 1$. Hence

$$\partial\tilde{A}^U{}_i/\partial\tilde{Q}^U{}_i = V_i - W_i\tilde{Q}^U - (1/M_i)\tilde{Q}^U{}_i - (1/M_i)\tilde{N}_{Ci} = 0$$

$$\Rightarrow \tilde{Q}^U{}_i = V_iM_i - \tilde{N}_{Ci} - W_iM_i\tilde{Q}^U, \text{ which on summing} \qquad \text{[A4.2]}$$

$$\Rightarrow \tilde{Q}^U = \Sigma(V_iM_i-\tilde{N}_{Ci}) - \Sigma W_kM_k\tilde{Q}^U$$

$$\Rightarrow \tilde{Q}^U = \Sigma(V_iM_i-\tilde{N}_{Ci}) / (1+\Sigma W_kM_k) \qquad \text{as in [2.71]}$$

$$\Rightarrow Q^U := \mathrm{E}[\tilde{Q}_U] = \Sigma V_iM_i/(1+\Sigma W_kM_k); \qquad \text{[A4.4]}$$

$$\Rightarrow \mathrm{E}[(\tilde{Q}_U)^2] = [\Sigma V_iM_i/(1+\Sigma W_kM_k)]^2 + \Sigma\,\mathrm{E}[\tilde{N}_{Ci}{}^2]/(1+\Sigma W_kM_k)^2$$

$$= [(\Sigma V_iM_i)^2+D_C]/(1+\Sigma W_kM_k)^2 = (Q^U)^2 + D_C/(1+\Sigma W_kM_k)^2; \qquad \text{[A4.5]}$$

$$\mathrm{E}[\tilde{N}_{Ci}\tilde{Q}_U] = -\,\mathrm{E}\,(\tilde{N}_{Ci})^2/(1+\Sigma W_kM_k) = -\,D_{Ci}/(1+\Sigma W_kM_k). \qquad \text{[A4.6]}$$

So from [2.70] and [A4.2],

$$\tilde{A}^U{}_i := V_i\tilde{Q}^U - ½W_i(\tilde{Q}^U)^2 - ½(1/M_i)(V_iM_i-\tilde{N}_{Ci}-W_iM_i\tilde{Q}^U)^2$$

$$- (1/M_i)(V_iM_i-\tilde{N}_{Ci}-W_iM_i\tilde{Q}^U)\tilde{N}_{Ci}; \qquad \text{as in [2.73]}$$

$$\Rightarrow A^U{}_i := \mathrm{E}[\tilde{A}^U{}_i]$$

$$= V_iQ^U - ½W_i[(Q^U)^2+D_C/(1+\Sigma W_kM_k)^2] - ½V_i^2M_i - ½(1/M_i)D_{Ci}$$

$$- ½W_i^2M_i[(Q^U)^2+D_C/(1+\Sigma W_kM_k)^2] + V_iW_iM_iQ^U$$

$$+ W_iD_{Ci}/(1+\Sigma W_kM_k) + (1/M_i)D_{Ci} - W_iD_{Ci}/(1+\Sigma W_kM_k)$$

$$= \bar{A}^U{}_i + F^U{}_i \text{ where} \qquad \text{as in [2.74]}$$

$$\bar{A}^U{}_i := V_iQ^U - ½W_i(1+W_iM_i)(Q^U)^2 - ½V_i^2M_i + V_iW_iM_iQ^U, \quad \text{as in [2.76]}$$

$$F^U{}_i = -\,½W_i(1+W_iM_i)[D_C/(1+\Sigma W_kM_k)^2] + ½(1/M_i)D_{Ci} \qquad \text{as in [2.77].}$$

---

9. We choose this maximand for simplicity for the risk-averse as well as the risk-neutral case. For formal consistency, under risk aversion i should maximise not its expected net benefit, but its expected payoff. However, the latter maximisation has no analytic solution and differs only very slightly from the former.

APPENDIX 5 PAYOFF FROM ET OR A TAX INSTEAD OF UNILATERALISM

From [3.2],

$$E[\exp\{-r(\varepsilon'\mathbf{F}^j_i\varepsilon+\mathbf{h}^j_i{}'\varepsilon)\}]$$

$$= \int_{\mathbb{R}^{4n}} \exp\{-r(\varepsilon'\mathbf{F}^j_i\varepsilon+\mathbf{h}^j_i{}'\varepsilon)\}\ (2\pi)^{-2n}|\mathbf{Z}|^{-½} \exp\{-½\varepsilon'\mathbf{Z}^{-1}\varepsilon\}\ d\varepsilon$$

$$= (2\pi)^{-2n}|\mathbf{Z}|^{-½} \int_{\mathbb{R}^{4n}} \exp\{-½[2r(\varepsilon'\mathbf{F}^j_i\varepsilon+\mathbf{h}^j_i{}'\varepsilon) + \varepsilon'\mathbf{Z}^{-1}\varepsilon]\ d\varepsilon$$

$$= (2\pi)^{-2n}|\mathbf{Z}|^{-½} \int_{\mathbb{R}^{4n}} \exp\{-½[\varepsilon'(\mathbf{Z}^{-1}+2r\mathbf{F}^j_i)\varepsilon + 2r\mathbf{h}^j_i{}'\varepsilon]\}\ d\varepsilon. \qquad [A5.1]$$

We will assume that r is small enough to assure the positive-definiteness of $(\mathbf{Z}^{-1}+2r\mathbf{F}^j_i)$, so that the integral in [A5.1] converges.

Now substitute

$$\mathbf{S}^j_i := \mathbf{Z}^{-1}+2r\mathbf{F}^j_i \qquad [A5.2]$$

Since $\mathbf{Z}$ and $\mathbf{F}^j_i$ are symmetric, so are $\mathbf{Z}^{-1}$, $2r\mathbf{F}^j_i$, $\mathbf{S}^j_i$ (= $\mathbf{S}^j_i{}'$) and $\mathbf{S}^{j\,-1}_i$ (= $(\mathbf{S}^{j\,-1}_i)'$). So using a device in Tallis (1961, p224),

$$(\varepsilon+r\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i)'\mathbf{S}^j_i(\varepsilon+r\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i) - r^2\mathbf{h}^j_i{}'\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i$$

$$= \varepsilon'\mathbf{S}^j_i\varepsilon + r\mathbf{h}^j_i{}'(\mathbf{S}^{j\,-1}_i)'\mathbf{S}^j_i\varepsilon + r\varepsilon'\mathbf{S}^j_i\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i + r^2\mathbf{h}^j_i{}'(\mathbf{S}^{j\,-1}_i)'\mathbf{S}^j_i\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i - r^2\mathbf{h}^j_i{}'\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i$$

$$= \varepsilon'\mathbf{S}^j_i\varepsilon + r\mathbf{h}^j_i{}'\mathbf{S}^{j\,-1}_i\mathbf{S}^j_i\varepsilon + r\varepsilon'\mathbf{h}^j_i + r^2\mathbf{h}^j_i{}'\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i - r^2\mathbf{h}^j_i{}'\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i$$

$$= \varepsilon'\mathbf{S}^j_i\varepsilon + 2r\mathbf{h}^j_i{}'\varepsilon \quad \text{since } \mathbf{h}^j_i{}'\varepsilon = \varepsilon'\mathbf{h}^j_i \text{ (both are scalars).} \qquad [A5.3]$$

Inserting [A5.2] and then [A5.3] into [A5.1] gives

$$E[\exp\{-r(\varepsilon'\mathbf{F}^j_i\varepsilon+\mathbf{h}^j_i{}'\varepsilon)\}]$$

$$= (2\pi)^{-2n}|\mathbf{Z}|^{-½} \int_{\mathbb{R}^{4n}} \exp\{-½(\varepsilon'\mathbf{S}^j_i\varepsilon+2r\mathbf{h}^j_i{}'\varepsilon\}\ d\varepsilon$$

$$= (2\pi)^{-2n}|\mathbf{Z}|^{-½} \int_{\mathbb{R}^{4n}} \exp\{½r^2\mathbf{h}^j_i{}'\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i - ½(\varepsilon+r\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i)'\mathbf{S}^j_i(\varepsilon+r\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i)\}\ d\varepsilon$$

$$= (|\mathbf{S}^j_i|/|\mathbf{Z}|)^{½} \exp(½r^2\mathbf{h}^j_i{}'\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i)$$

$$\times \int_{\mathbb{R}^{4n}} d\varepsilon\ \exp\{-½(\varepsilon+r\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i)'\mathbf{S}^j_i(\varepsilon+r\mathbf{S}^{j\,-1}_i\mathbf{h}^j_i)\} / (2\pi)^{2n}|\mathbf{S}^j_i|^{½}.$$

The integral here is now that of an multinormal probability density function, with dispersion matrix $\mathbf{S}^{j}_{i}{}^{-1} = (\mathbf{Z}^{-1}+2r\mathbf{F}^{j}_{i})^{-1}$ instead of $\mathbf{Z}$, covering the whole of $4n$-dimensional space (displaced by having $\varepsilon+r\mathbf{S}^{j}_{i}{}^{-1}\mathbf{h}^{j}_{i}$ instead of ε, but this makes no difference since the whole space is integrated over). So the integral equals 1, leaving

$$\mathrm{E}[\exp\{-r(\varepsilon'\mathbf{F}^{j}_{i}\varepsilon+\mathbf{h}^{j}_{i}{}'\varepsilon)\}] = (|\mathbf{S}^{j}_{i}|/|\mathbf{Z}|)^{½} \exp(½r^2\mathbf{h}^{j}_{i}{}'\mathbf{S}^{j}_{i}{}^{-1}\mathbf{h}^{j}_{i}), \text{ so} \qquad [A5.4]$$

$$\begin{aligned}\mathrm{E}[\exp\{-r(\tilde{A}^{j}_{i}-A^{U}_{i})\}] &= \exp\{-r(\bar{A}^{j}_{i}-A^{U}_{i})\}\ \mathrm{E}[\exp\{-r(\varepsilon'\mathbf{F}^{j}_{i}\varepsilon+\mathbf{h}^{j}_{i}{}'\varepsilon)\}] \\ &= (|\mathbf{S}^{j}_{i}|/|\mathbf{Z}|)^{½} \exp\{½r^2(\mathbf{h}^{j}_{i})'(\mathbf{S}^{j}_{i})^{-1}\mathbf{h}^{j}_{i}-r(\bar{A}^{j}_{i}-A^{U}_{i})\}\end{aligned}$$

as in [3.17f].

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Referees' Appendix 1 Notation and Acronyms

Dimensions (units of measurement) are shown in square brackets; [1] indicates a dimensionless number; equation numbers show where the term is introduced.

a

$\tilde{A}^j_i$	:= $\tilde{B}^j_i - \tilde{C}^j_i$ = party *i*'s dollar-valued net benefit (*A*dvantage) from abatement under policy mechanism *j* [$/yr]	[2.34]
A^j_i	:= $E[\tilde{A}^j_i] = \bar{A}^j_i + F^j_i$	[3.9]
$\bar{A}^j_i$	party *i*'s net benefit from policy *j* under certainty	[3.4]
A^U_i	party *i*'s dollar-valued net benefit (*A*dvantage) from unilateral abatement by all parties [$/yr]	[3.33]
b	billion (= 10^9 = giga = G)	
^{b}	denotes *b*usiness-as-usual	

b

BAU	Business-As-Usual	
B	global *B*enefit of global abatement [$/yr]	
$\tilde{B}^j_i$	:= $V_i\tilde{Q}^j - ½W_i(\tilde{Q}^j)^2$ = party *i*'s *B*enefit from global abatement using mechanism *j* [$/yr]	[2.38]

c

$\tilde{C}^j_i$	:= $½(1/M_i)Q^j_i{}^2 + Q^j_i\varepsilon_{Ci} - \tilde{R}^j_i$ = party *i*'s *C*ost (net of emissions trading) of its own abatement using mechanism *j* [$/yr]	[2.41]
CO_2	carbon dioxide	
d	differential symbol	
D_{Ci}	:= $M_i^2\sigma_{Ci}^2$	[2.46]
D_{Ei}	:= $E^b_i{}^2[(\alpha_i - \beta_i x_i)^2\sigma_{Yi}^2 + \alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]$ [t^2/yr^2]	[2.17]
D^0_{Ei}	:= $E^b_i{}^2[\alpha_i^2\sigma_{Yi}^2 + \alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]$ [t^2/yr^2]	[2.22]
D^*_{Ei}	:= $E^b_i{}^2[\alpha_i^2\sigma_{\eta i}^2 + (1-\alpha_i)^2\sigma_{\rho i}^2]$ [t^2/yr^2]	[2.24]
e	the number e	
E[.]	Expectation operator	
$\tilde{E}^j_i$	party *i*'s actual *E*missions under abatement mechanism *j* [t/yr]	[2.3]
E^b_i	party *i*'s expected*b*usiness-As-Usual (unabated) *E*missions [t/yr]	[2.4]

$\tilde{E}^b_i$	party i's realised *b*usiness-As-Usual (unabated) *E*missions [t/yr]	[2.4]
ET	(inter-party) Emissions Trading	
$f(.)$	arbitrary function	
$\tilde{F}^j_i$	part of realised net benefit $\tilde{A}^j_i$ containing squared and cross errors	[3.6]
future	in 2020	
g		
G	giga (= 10^9 = billion)	
G^j	expected global gain from mechanism j	
$\tilde{G}^j_i$	party i's realised *G*ain from mechanism j := $\tilde{A}^j_i - \tilde{A}^U_i$	
GDP	Gross Domestic Product	
GHG	GreenHouse Gas(es)	
h		
$_h$	index in summation over total of m errors	
H^T_{Ci}	:= $(p^T M_i - E^b_i + X^T_i)/M$ \$/t	[3.8]
H^T_{Ei}	:= $(p^T M_i - E^b_i + X_i)/M + V_i - W_i pM$ \$/t	[3.7]
$H^\$_{Ci}$	:= $(X^\$_i/X^\$)p^\$ - (V_i - W_i p^\$ M)$.	[3.32]
$H^\$_{Ei}$	:= $(X^\$_i/X^\$)p^\$$, and	[3.31]
i	one of n or $n-1$ parties (never a single unilateralist or NBT holder)	
$_i$	denotes party i (used with n or $n-1$ parties)	
$_{-i}$	denotes sum over all parties except i	
I	arbitrary integral	
^{j}	denotes one of three types of abatement mechanism ($^j = {}^T, {}^\$, {}^@$)	
J	arbitrary constant, limit of integration or (with $\tilde{}$) random variable	
k	one of n parties	
$\tilde{K}^\$_i$	gross tax revenue to party i	[2.29]
l		
L_i	population of party i [person]	
m	arbitrary number of errors (generally = $4n$, since each of n parties has 3+1 uncertainties)	
m_i	:= M_i/E^b_i, relative abatement potential [t/\$]	

M	mega (= 10^6 = million)	
MAC	Marginal Abatement Cost	
M_i	party *i*'s absolute abatement potential (inverse slope of *M*AC curve) [t^2/$.yr]	[2.41]
n	full number of parties included in abatement policy [1]	
$\tilde{N}_{Ci}$	$:= M_i\varepsilon_{Ci}$ = weighted marginal abatement cost uncertainty [$/t]	[2.41]
$\tilde{N}_{Ei}$	$:= [(\alpha_i-\beta_i x_i)\varepsilon_{Yi}+\alpha_i\varepsilon_{\eta i}+(1-\alpha_i)\varepsilon_{\rho i}]\ E^b_i$.	[2.16]
$\tilde{N}^0_{Ei}$	$:= [\alpha_i\varepsilon_{Yi}+\alpha_i\varepsilon_{\eta i}+(1-\alpha_i)\varepsilon_{\rho i}]\ E^b_i$	[2.21]
$\tilde{N}^*_{Ei}$	$:= [\alpha_i\varepsilon_{\eta i}+(1-\alpha_i)\varepsilon_{\rho i}]\ E^b_i$	[2.23]
p	expected world emissions *p*rice or shadow price [$/t]	[2.46]
$\bar{p}$	$= V/(1+WM)$, emissions price in risk-neutral case [$/t]	[2.67]
P		
q	generic quantity	
$\tilde{Q}^j_i$	party *i*'s realised abatement under mechanism *j* [Gt/yr]	[2.3]
r	risk aversion parameter (curvature of payoff function) [yr/$]	[2.34]
$\tilde{R}^j_i$	party *i*'s trading revenue from abatement mechanism *j* [$/yr]	[2.27]
s	arbitrary function argument, or integration variable	
t	tonne of CO_2-equivalent of GHG	
t	*t*ime (used for cumulative emission responsibility) [yr]	
^{T}	denotes Emissions *T*rading	[2.26]
$\tilde{U}^j_i$	*P*ayoff from *i*'s $-valued gain from abatement mechanism *j* [$/yr]	[2.36]
^{U}	denotes global unilateral abatement scenario	sec 2.3(iii)
V_i	party *i*'s linear *V*alue per unit of global abatement $\tilde{B}_i$ [$/t]	[2.38]
w		
W_i	local (downward) slope of marginal benefit [$.yr/G($t^2$)]	[2.38]
x_i	$:= X_i/E^b_i$, party *i*'s expected absolute emissions target as proportion of expected BAU emissions [1]	[2.8]
X_i	party *i*'s expected absolute emissions target [t/yr]	[2.9]
$\tilde{X}_i$	$:= x_i E^b_i(1+\beta_i\varepsilon_{Yi})$ = party *i*'s realised, absolute (but uncertain) emissions target under flexible tradable permits [t/yr]	[2.8]
y_i	$:= Y_i/L_i$, party *i*'s expected, future per capita GDP	
yr	year	

Y_i	party i's expected future GDP [\$/yr]	[2.12]
$\tilde{Y}_i$	$= Y_i(1+\varepsilon_{Yi})$ = party i's realised future GDP [\$/yr]	[2.12]
z_i	multiplier of risk-averse part of payoff	[2.36]
Z		
α_i	proportion of economy i in which emissions are GDP-linked	[2.4]
β_i	degree to which i's emissions target is indexed to GDP [1]	[2.8]
β_i^*	optimal β_i (that maximises i's expected payoff, given fixed x_i) [1]	[2.13]
Γ^T_i	$:= (p^T-H_{Ei})^2 D_{Ei} + (H_{Ei})^2 \Sigma_{-i} D_{Ek} + (p^T-H_{Ci})^2 D_{Ci} + (H_{Ci})^2 \Sigma_{-i} D_{Ck}$.	[3.19]
$\Gamma^\$_i$	$:= (p^\$-H^\$_{Ei})^2 D^0_{Ei} + (H^\$_{Ei})^2 \Sigma_{-i} D^0_{Ek} + (p^\$-H^\$_{Ci})^2 D_{Ci} + (H^\$_{Ci})^2 \Sigma_{-i} D_{Ck}$.	[3.41]
γ		
Δ		
δ		
ε_{Ci}	absolute random part of $\tilde{C}_i$ [\$/yr]	[2.41]
ε_{Yi}	BAU emissions error caused by GDP fluctuations [1]	[2.4]
$\varepsilon_{\eta i}$	BAU emissions error caused by intensity fluctuations [1]	[2.4]
$\varepsilon_{\rho i}$	BAU emissions error caused by other fluctuations [1]	[2.4]
ζ		
η^b_i	party i's expected BAU emissions intensity $= E^b_i/Y$ [t/S]	
Θ		
θ	arbitrary coefficient [1]	
κ		
$\tilde{\Lambda}^T_i$	$= \bar{A}^T_i + F^T_i - p^T(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + H^T_{Ei}\tilde{N}_E + H^T_{Ci}\tilde{N}_C$	[3.14]
$\tilde{\Lambda}^\$_i$	$= \bar{A}^\$_i + F^\$_i - p^\$(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + H^\$_{Ei}\tilde{N}_E + H^\$_{Ci}\tilde{N}_C$	[3.36]
λ		
μ	arbitrary coefficient [1]	
ν		
Ξ		
ξ		
Π	product symbol	
π	the number pi, as in normal distributions	

ρ denotes part of economy where emissions are independent of GDP and intensity [3.5]

Σ summation symbol, over i or $k = 1,...,n$

Σ_{-i} summation over all $k \neq i$

σ_{Ci} standard deviation of absolute uncertainty in i's MAC [$/t] [2.42]

σ_{Yi} standard deviation of proportional uncertainty in i's GDP [1] [2.7]

$\sigma_{\eta i}$ standard deviation of proportional uncertainty in i's intensity [1] [2.7]

$\sigma_{\rho i}$ standard deviation of proportional uncertainty in i's non-GDP-linked emissions [1] [2.7]

σ_μ^2 $\Sigma_h \mu_h^2 \sigma_h^2$ (hence $\sigma_\theta^2 = \Sigma_h \theta_h^2 \sigma_h^2$) [A5.3]

τ variable of integration for expected payoff

υ

Φ

ϕ

χ

Ψ

ψ

Ω

ω

$\tilde{}$ denotes random or uncertain variable (with $J := \mathrm{E}[\tilde{J}]$ for any $\tilde{J}$)

$\bar{}$ denotes expectation of equivalent variable under certainty

$ US dollar amount in constant 2000 dollars

$^{\$}$ denotes a Tax with Thresholds [2.28]

$^{@}$ denotes Non-Tradable Targets (NTTs) [2.33]

^{0} (as qualifier of ^{T} or $^@$) denotes no indexation of threshold/target [2.21]

* (as qualifier of ^{T} or $^@$) denotes optimal indexation of threshold/target [2.33]

Referees' Appendix 2 Algebra for Emissions Trading

From [A1.3]:

$$\tilde{A}^T{}_i = V_i(p^TM+\tilde{N}_E) - ½W_i(p^TM+\tilde{N}_E)^2 + \tilde{p}^T(\tilde{p}^TM_i-\tilde{N}_{Ci}-E^b{}_i+X^T{}_i-\tilde{N}_{Ei})$$
$$- ½(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 - (1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})\tilde{N}_{Ci}$$
$$= V_ip^TM - ½W_i(p^T)^2M^2 + (V_i-W_ip^TM)\tilde{N}_E - ½W_i\tilde{N}_E{}^2$$
$$+ (1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 + (1/M_i)\tilde{N}_{Ci}(\tilde{p}^TM_i-\tilde{N}_{Ci}) - \tilde{p}^T(E^b{}_i-X^T{}_i+\tilde{N}_{Ei})$$
$$- ½(1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})^2 - (1/M_i)(\tilde{p}^TM_i-\tilde{N}_{Ci})\tilde{N}_{Ci}$$
$$= V_ip^TM - ½W_i(p^T)^2M^2 + (V_i-W_ip^TM)\tilde{N}_E - ½W_i\tilde{N}_E{}^2$$
$$+ ½(1/M_i)[(p^TM+\tilde{N}_E+\tilde{N}_C)M_i/M-\tilde{N}_{Ci}]^2 - [(p^TM+\tilde{N}_E+\tilde{N}_C)/M](E^b{}_i-X^T{}_i+\tilde{N}_{Ei})$$
$$= V_ip^TM - ½(p^T)^2W_iM^2 - p^T(E^b{}_i-X^T{}_i)$$
$$+ [V_i-W_ip^TM-(E^b{}_i-X^T{}_i)/M]\tilde{N}_E - ½W_i\tilde{N}_E{}^2$$
$$+ ½(p^TM+\tilde{N}_E+\tilde{N}_C)^2M_i/M^2 - \tilde{N}_{Ci}(p^TM+\tilde{N}_E+\tilde{N}_C)/M + ½(1/M_i)\tilde{N}_{Ci}{}^2$$
$$- p^T\tilde{N}_{Ei} - (\tilde{N}_C/M)(E^b{}_i-X^T{}_i) - [(\tilde{N}_E+\tilde{N}_C)/M]\tilde{N}_{Ei}$$
$$= V_ip^TM + ½(p^T)^2(M_i-W_iM^2) - p^T(E^b{}_i-X^T{}_i)$$
$$+ [p^TM_i/M+V_i-W_ip^TM-(E^b{}_i-X^T{}_i)/M]\tilde{N}_E + ½(M_i/M^2-W_i)\tilde{N}_E{}^2$$
$$+ (p^TM_i/M)\tilde{N}_C + (M_i/M^2)(\tilde{N}_E\tilde{N}_C+½\tilde{N}_C{}^2) - p^T\tilde{N}_{Ci} - (1/M)(\tilde{N}_E+\tilde{N}_C)\tilde{N}_{Ci}$$
$$+ ½(1/M_i)\tilde{N}_{Ci}{}^2 - p^T\tilde{N}_{Ei} - [(E^b{}_i-X^T{}_i)/M]\tilde{N}_C - (1/M)\tilde{N}_{Ei}(\tilde{N}_E+\tilde{N}_C)$$
$$= \bar{A}^T{}_i + H^T{}_{Ei}\tilde{N}_E + H^T{}_{Ci}\tilde{N}_C - p^T(\tilde{N}_{Ei}+\tilde{N}_{Ci})$$
$$+ ½(M_i/M^2-W_i)\tilde{N}_E{}^2 + ½(M_i/M^2)\tilde{N}_C{}^2 + ½(1/M_i)\tilde{N}_{Ci}{}^2$$
$$+ (M_i/M^2)\tilde{N}_E\tilde{N}_C - (1/M)\tilde{N}_{Ci}{}^2 - (1/M)\tilde{N}_{C-i}\tilde{N}_{Ci} - (1/M)\tilde{N}_E\tilde{N}_{Ci}$$
$$- (1/M)\tilde{N}_{Ei}{}^2 - (1/M)\tilde{N}_{E-i}\tilde{N}_{Ei} - (1/M)\tilde{N}_{Ei}\tilde{N}_C, \text{ hence}$$

$$\tilde{A}^T{}_i = \bar{A}^T{}_i + \tilde{F}_i - p^T(\tilde{N}_{Ei}+\tilde{N}_{Ci}) + H^T{}_{Ei}\tilde{N}_E + H^T{}_{Ci}\tilde{N}_C \qquad \text{as in [3.3]}$$

where $\bar{A}^T{}_i := V_ip^TM + ½(p^T)^2(M_i-W_iM^2) - p^T(E^b{}_i-X^T{}_i)$ \$/yr, as in [3.4]

$$\tilde{F}_i := ½(M_i/M^2-W_i)\tilde{N}_E{}^2 - (1/M)\tilde{N}_{Ei}{}^2 + ½(M_i/M^2)\tilde{N}_C{}^2 + [½(1/M_i)-1/M]\tilde{N}_{Ci}{}^2$$
$$+ (M_i/M^2)\tilde{N}_E\tilde{N}_C - (1/M)[\tilde{N}_{Ci}(\tilde{N}_{C-i}+\tilde{N}_E)+\tilde{N}_{Ei}(\tilde{N}_{E-i}+\tilde{N}_C)] \qquad \text{as in [3.6]}$$

$H^T{}_{Ei} := (p^TM_i-E^b{}_i+X^T{}_i)/M + V_i - W_ip^TM$, and as in [3.7]

$H^T{}_{Ci} := (p^TM_i-E^b{}_i+X^T{}_i)/M$, as in [3.8].