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SUSTAINED GROWTH FROM NON-RENEWABLE RESOURCES:
CONSTANT ABSOLUTE GENUINE SAVINGS AND
CONSTANT RELATIVE GENUINE SAVINGS COMPARED

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ANO (2005) showed the interesting theoretical result that keeping a constant *absolute* level of genuine savings (GS) in a constant-returns-to-scale (CRS), Dasgupta-Heal economy results in sustained development with ever-rising consumption if the level of GS is positive. This is in contrast to the single-peaked development followed by the optimal path of this economy with a constant utility discount rate (Dasgupta and Heal 1979, Pezzey and Withagen 1998). This note compares ANO's constant absolute genuine savings (CAGS) path with the 'hyperbolic' development path in Pezzey (2004), which entails a constant *relative* level of GS (CRGS) − relative to output, consumption and investment − and also has ever-rising consumption if that level is positive. There are many similarities and differences, and to clarify them I consider only a simplified version of the CRGS path with the same production function as in ANO (2005). So here we have no technical progress and CRS in production: $F = K^\alpha R^\beta$, $\alpha+\beta = 1$, instead of $F = K^\alpha R^\beta(1+\theta_0 t)^\nu$, $\alpha+\beta \leq 1$ as in Pezzey (2004).

ANO's CAGS path is only the third known exact algebraic solution of the Dasgupta-Heal economy with constant technology, the other two being Pezzey and Withagen (1998, Section V), and CRGS with $\nu = 0$. Key algebraic formulae for CAGS, some given by ANO and others derived in an Appendix, are given in ANO's notation in Table 1, along with corresponding formulae for the simplified CRGS economy. An important caveat is that though the derived parameter θ_0 has the same formula in both economies, its levels differ because they depend on $R(0)$, initial resource extraction. The levels of $R(0)$ generally differ because they depend on the GS parameters, respectively $\bar{G}$ (absolute) and g (relative), both of which are essentially arbitrary choices. An explicit expression for $R(0)$ and hence θ_0 in terms of parameters α, g, S_0 and K_0 can be derived from $-\dot{S}(0)$ under CRGS. However, this

Table 1 Comparing paths of Constant Absolute and Constant Relative Genuine Savings in a CRS Dasgupta-Heal economy

Quantity	Constant Absolute Genuine Savings (ANO 2005; N.B. $\alpha+\beta = 1$)	Constant Relative Genuine Savings (Pezzey 2004 with $\alpha+\beta = 1$, $\nu = 0$, $1-\rho\beta/\alpha = g$)
Inputs, outputs, resource depletion, initial stocks	$[K(t)]^{\alpha}[R(t)]^{\beta} = F(t) = C(t) + \dot{K}(t)$; $\dot{S}(t) = -R(t)$; $K(0) = K_0$, $S(0) = S_0$	
Genuine saving $G(t) := \dot{K}(t) - F_R(t)R(t)$	$\bar{G}$	$g\,F(t)$
Consumption constant if:	$\bar{G} = 0$	$g = 0$
Feasibility constraint	$\bar{G} < \alpha F(0)$	$g < \alpha-\beta$
Derived parameter θ_0	$\beta[R(0)/K_0]^{\beta}$	$\beta[R(0)/K_0]^{\beta} = \beta[(\alpha-\beta-g)S_0/K_0]^{\beta/\alpha}$
Output $F(t)$	$(\bar{G}/\beta)\ln(1+\theta_0 t) + F(0)$	$(1+\theta_0 t)^{g/\beta}\,F(0)$
Output growth $\dot{F}(t)/F(t)$	$\theta_0\bar{G}\ /\ \beta(1+\theta_0 t)F(t)$	$\theta_0 g\ /\ \beta(1+\theta_0 t)$
NNP $Y(t) = C(t) + G(t)$	$\alpha F(t)$	$\alpha F(t)$
Consumption $C(t)$	$\alpha F(t) - \bar{G}$	$(\alpha-g)F(t)$
Investment $\dot{K}(t)$	$\beta F(t) + \bar{G}$	$(\beta+g)F(t)$
Capital stock $K(t)$	$[(1+\theta_0 t)\beta/\theta_0]\ F(t)$	$[(1+\theta_0 t)\beta/\theta_0]\ F(t)$
Interest rate $r(t) = \alpha[R(t)/K(t)]^{\beta}$	$\theta_0\alpha/\beta(1+\theta_0 t)$	$\theta_0\alpha/\beta(1+\theta_0 t)$
Resource flow $R(t)$	$[(1+\theta_0 t)\beta/\theta_0]^{-\alpha/\beta}\ F(t)$	$[(1+\theta_0 t)\beta/\theta_0]^{-\alpha/\beta}\ F(t)$
Resource stock $S(t)$	$S_0 - \int_0^t R(x)dx$	$S_0(1+\theta_0 t)^{-(\alpha-\beta-g)/\beta}$
Initial wealth $W(0)$	$[\alpha/(\alpha-\beta) + \bar{G}/\beta F(0)]\ K_0$	$[(\alpha-g)/(\alpha-\beta-g)]\ K_0$
Max. initial wealth	$[\alpha/(\alpha-\beta) + \alpha/\beta]\ K_0$	∞

is not possible for CAGS, because $-\dot{S}(0)$ contains θ_0, binding θ_0 and $R(0)$ together in an implicit formula. (The explicit formula for $S(t)$ is not shown in Table 1 for CAGS, because although $\int_0^t R(x)dx$ can be integrated by parts, it yields a complex expression of no great interest.)

Note that $\bar{G}$ and g play some similar roles. When they are zero, the economy reverts to the Solow (1974) constant consumption path; when they are positive, consumption grows forever; but they must not exceed a feasibility constraint ($\bar{G} < \alpha F(0)$ under CAGS, and $g < \alpha - \beta$ under CRGS). Not noted by ANO (2005) is that shrinking economies, with either $\bar{G}$ or $g < 0$, are feasible solutions. However, under CAGS, $G = \bar{G} < 0$ becomes an increasingly insupportable burden and the economy collapses in finite time; while under CRGS, $G = gF$ is always supportable, so the economy declines to nothing only asymptotically.

The most important difference between the economies is in long run growth rates. Intuitively, holding GS absolutely constant will get easier as time passes in a growing economy, and GS becomes smaller relative to output. Holding GS constant relative to output, however, gets no easier (or harder) in this sense as time passes. CRGS is thus a bigger long run commitment to saving and investment than CAGS, and can be expected to give higher long run growth. Algebraically, this is confirmed by output $F(t)$ being a logarithmic function of $(1+\theta_0 t)$ under CAGS, but a power function of $(1+\theta_0 t)$ under CRGS. The growth rate $\dot{F}/F$ declines faster in CAGS; and while CAGS output will always be strictly concave, CRGS output can be strictly convex if $g > \beta$, which is possible if $\alpha > 2/3$.

A notable similarity between the two economies is that they share identical formulae for their NNPs, their capital/output ratios K/F (and hence interest rate $r = \alpha F/K$), and their resource/output ratios R/F. However, because of the dependence of θ_0 on the choice parameters $\bar{G}$ or g, these ratios will differ numerically.

Finally, both levels of initial wealth $W(0)$ are independent of the initial resource stock. But $W(0)$ is bounded as $\bar{G} \rightarrow \alpha F(0)$ (the feasibility constraint) in the CAGS case, while it is unbounded as $g \rightarrow \alpha - \beta$ under CRGS, reflecting the latter's higher long term growth rate.

References

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Appendix

Extra results for CAGS solution path

The result $C = \alpha F - \bar{G}$ is from (A3) in ANO. Inverting it and substituting for C from (A12) gives the result for F. r is a rearrangement of (A7). K comes from $\alpha F/r$, and R is then $(r/\alpha)^{1/\beta}K$. $W(0)$ is a rearrangement of (A11). Other quantities follow readily from their definitions and the above results.

Extra results for CRGS solution path

These mostly follow directly from results (7)-(10) in Pezzey (2004) by putting $\nu = 0$, using $\alpha+\beta = 1$, hence for example $\xi+\sigma = \alpha/\beta$, and defining $g := 1-\rho\beta/\alpha$. The exceptions are:

$G = \dot{K}-F_R R = \alpha F-C = [\alpha-(\rho-\alpha)\beta/\alpha]F = (1-\rho\beta/\alpha)F = gF$, and

$W(0) = K_0 + F_R(0)S_0$ (thanks to CRS, we can use this rather than the equivalent but less direct definition on p475 of Pezzey 2004)

$= K_0 + \beta S_0(\beta/\theta_0)^{\alpha/\beta}$

$= K_0 + \beta^{1+\alpha/\beta}S_0\{\beta^{1/\alpha}[(\rho/\alpha-2)S_0/K_0]^{\beta/\alpha}\}^{-\alpha/\beta}$

$= K_0 + K_0[(1-2\beta-g)/\beta]^{-1}$

$= [(\alpha-g)/(\alpha-\beta-g)]\ K_0$, as in Table 1.