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Measuring Technical Progress in Gross and Net Products

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Abstract On the optimal path of an economy with capital and non-renewable resource inputs, and constant returns output of consumption and investment, the rate of exogenous technical progress in net national product equals the rate of progress in (gross) production, divided by one minus the production elasticity of the resource flow.

Keywords. Exogenous technical progress, net national product, non-renewable resources

JEL classification: O30, Q30

1. The result

Consider a closed, smooth, representative-agent economy with a stock of human-made capital $K(t)$ at time t, a stock of a non-renewable resource $S(t)$, and an aggregate production function $F(K,R,t)$ that depends on the capital stock, resource depletion flow $R(t)$, and time (that is, it exhibits exogenous technical progress). Production is divided, with constant returns to scale but not necessarily linearly, into investment $\dot{K}(t)$ and consumption $C(t)$. So we can write, using overdots to denote total time derivatives and subscripts to denote partial derivatives as usual,

$$\dot{K} = J[F(K,R,t),C]; \quad K(0) = K_0 > 0; \quad J_F > 0,\ J_C < 0 \qquad [1]$$

where $J[.]$ is itself a constant returns function of F and C, hence

$$J = FJ_F + CJ_C. \qquad [2]$$

The dynamics of the non-renewable resource are given by

$$\dot{S} = -R; \quad S(0) = S_0 > 0. \qquad [3]$$

The economy chooses consumption and resource flow over time to maximise, using some discount factor $\phi(t)$, the present discounted value of $U(C)$, the instantaneous utility of consumption (with $U_C > 0$, $U_{CC} < 0$):

$$\max_{C,R} \int_0^\infty \phi(t)U[C(t)]dt; \quad \phi(t) > 0,\ \dot{\phi}(t) < 0,\ \phi(0) = 1,\ \lim_{t\to\infty}\phi(t) = 0. \qquad [4]$$

We assume the convergence of the integral, and the existence and uniqueness of a solution to [4], called the optimal path.

The current value Hamiltonian of [4] is

$$H = U(C) + \mu^K\dot{K} + \mu^S\dot{S} = U(C) + \mu^K J[F(K,R,t),C] - \mu^S R \qquad [5]$$

The maximum principle for an interior solution gives

$$H_C = U_C + \mu^K J_C = 0 \quad \Rightarrow \quad \mu^K = -U_C/J_C \qquad [6]$$

$$H_R = \mu^K J_F F_R - \mu^S = 0 \quad \Rightarrow \quad \mu^S = \mu^K J_F F_R = -U_C J_F F_R/J_C \qquad [7]$$

Net national product (NNP) is defined as

$$Y := C + (\mu^K/U_C)\dot{K} + (\mu^S/U_C)\dot{S}, \qquad [8]$$

$$= C - J/J_C - (J_F F_R/J_C)(-R) \quad \text{by [1], [3], [6] and [7],}$$

$$= (F - F_R R)J_F / (-J_C) \quad \text{by [2],}$$

so $$Y = (1-\varepsilon_R)FJ_F / (-J_C), \qquad [9]$$

where $\varepsilon_R := F_R/(F/R)$ is the production elasticity of the resource flow.

A fuller writing of [8] is

$$Y(C,K,S,R,t) = C + (\mu^K/U_C)J[F(K,R,t),C] - (\mu^S/U_C)R$$

so the exogenous effect of time on NNP is

$$Y_t(C,K,S,R,t) = (\mu^K/U_C)J_F F_t = J_F F_t / (-J_C) \quad \text{by [6]} \qquad [10]$$

Dividing [10] by [9] and rearranging gives our result: *the rate of exogenous technical progress in net national product is the rate of progress in (gross) production, divided by one minus the production elasticity of the resource flow:*

$$Y_t/Y = (F_t/F) \,/\, (1-\varepsilon_R). \qquad [11]$$

Intuitively, the pure time effect in $F(K,R,t)$ gives exogenous progress only in producing C and $\dot{K}$, but not in the resource rent $(\mu^S/U_C)R$. So the progress rate in making NNP = $C + \mu^K\dot{K} - \mu^S R$ is higher, by a factor $1/(1-\varepsilon_R)$ that shows the importance of the resource in production. If one relaxes the simplifying conditions assumed here (for example, by allowing multiple capitals or resources, or resource renewal), then the result obtained is usually not analytic, and if analytic not as neat as [11]. But the conclusion remains that technical progress in (gross) production generally does not occur at the same rate as technical progress in NNP on an optimal path.

2. The connection to Weitzman's technical progress premium

There is a connection between result [11] and the "technical progress premium" (TPP), defined by Weitzman (1997) as:

$$\text{TPP}(0) := [\Psi(0)/Y(0)] - 1, \qquad [12]$$

where $\Psi(t)$ is variously called "consumption NNP" (Asheim 1997); "sustainability" (by Weitzman); and "wealth-equivalent income" (Asheim 2000), the term we will use here. It is the consumption level which, if held constant forever, gives the same present discounted value of consumption as actual consumption on the optimal path. If the economy maximises welfare as in [4] and the interest rate is a constant r, it equals

$$\Psi(t) = r\int_t^\infty C(s)e^{-r(s-t)}ds. \qquad [13]$$

Weitzman (1997) showed that in this case, if there is exogenous technical progress (a pure effect of time) in producing NNP, then Y must be adjusted upwards to give an accurate measure of wealth-equivalent income Ψ, and the TPP in [12] that makes this adjustment is:

$$\text{TPP}(0) = \chi(0) \,/\, [r - g(0)], \quad \text{where} \qquad [14]$$

$$g(0) := \int_0^\infty \dot{Y}(t)e^{-rt}dt \,/\, \int_0^\infty Y(t)e^{-rt}dt, \quad \text{and} \qquad [15]$$

$$\chi(0) := \int_0^\infty Y_t(t)e^{-rt}dt \,/\, \int_0^\infty Y(t)e^{-rt}dt = Y_t(0)/Y(0) \text{ if } Y_t/Y \text{ is constant.} \qquad [16]$$

Here, $g(0)$ is the average future growth rate of NNP; while Weitzman describes $\chi(0)$ as "...the average future growth rate of the 'residual', which captures the pure effect of time alone on enhancement of productive capacity" (p7), or the "annual growth rate of total factor productivity" (p11).

The connection with our result is that one cannot replace Y_t and Y in expression [16] for χ with figures for F_t and F, which are relevant to total production of consumption and investment goods, but leave out the effect of resource depletion. Otherwise, from [11], the TPP will be underestimated.

One can get a rough idea of the significance of this theoretical result (but only by entering the realm of empirical controversy about long run production functions) by assuming a Cobb-Douglas form of $F(.)$ with exponential technical progress, following Stiglitz (1974) except with a constant labour supply implicitly assumed:

$$F(K,R,t) = K^{\alpha}R^{\beta}e^{\nu t}, \quad 0 < \alpha, \beta, \alpha+\beta \leq 1; \quad \nu > 0. \qquad [17]$$

Then, independently of the precise form of the consumption-investment frontier corresponding to $J(F,C)$ in [1],[1] result [11] becomes

$$Y_t/Y = \nu/(1-\beta). \qquad [18]$$

So using just the technical progress parameter ν from the (gross) production function [17] − which would be the rough effect of using data in our model for conventionally measured NNP = $C+\dot{K}$, rather than for "green" NNP = $C+\dot{K}-F_R R$ − would cause an underestimate of the TPP by a factor $(1-\beta)$.

According to Weitzman (1997, p11), this result would have no empirical significance in the United States now because "the depletion of natural capital like subsoil minerals, forests, or topsoil...is currently a neglible fraction of national product" (that is, any β-like parameter is tiny). But the relevance of such data, and the production functions for which they are estimated, to other countries and for many decades into the future can be questioned. In some cases, β-like parameters may be significant, and serious underestimates of the TPP may happen. Somewhat paradoxically, the more important that non-renewable resources are in production, the more important it is then to include exogenous technical progress in national accounts, in order to find a more accurate measure of wealth-equivalent income.

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1. For example, one could have $\dot{K} = F-C$, as in Stiglitz (1974) and most economic growth theory; or $\dot{K} = (F^{\eta}-C^{\eta})^{1/\eta}$, $\eta > 1$, which would be a specific example of the strictly concave trade-off between consumption C and investment $\dot{K}$ assumed by Weitzman (1976, p160).