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The Effect of Subsistence on Collapse and Institutional Adaption in Population-resource Societies

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The effect of subsistence on collapse and institutional adaptation in population-resource societies

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ABSTRACT: We extend the Brander-Taylor model of population and resource development in an isolated society by adding a resource subsistence requirement to people's preferences. This improves plausibility; amplifies population overshoot and collapse, and makes the steady state less stable; and allows for complete cessation of non-harvesting activities, in line with archaeological evidence for many societies. We then use bifurcation techniques to give a global analysis of four types of institutional adaptation: an ad valorem resource tax, and quotas on total resource harvest, total harvest effort and per capita effort. In all cases we find that a higher subsistence requirement makes it harder, or often impossible, for adaptation to avoid overshoot and collapse.‡

(JEL Q20, N57, J10)

KEYWORDS: population, renewable resources, subsistence, bifurcation, conservation

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1 Introduction

Many societies that depend mainly on renewable natural resources have risen and then fallen over several centuries. Why they did this is a question that has long interested anthropologists, archaeologists and economic historians (see for example Tainter (1988), Huntington (1917)). It has recently attracted attention from ecological-economic modellers, with the publication by Brander and Taylor (1998) (hereafter BT) of a model of Malthusian (nutrition-determined) population growth and open access harvesting of a renewable resource, applied to Easter Island in the South-East Pacific during A.D. 400-1900. In its heyday, the Easter Island civilization erected scores of huge stone statues, but what greeted the first European explorers to visit in the eighteenth century was a much reduced population living a bare subsistence existence. A key but difficult question raised by BT was why there was no institutional adaptation to prevent this overshoot and collapse, since there are many examples of open-access systems where common property management regimes have been developed to prevent the type of degradation observed in the BT model (Ostrom, 1990).

Recent developments of the BT model by Dalton and Coats (2000), Erickson and Gowdy (2000) and Reuveny and Decker (2000) have explored various institutional adaptations that could have prevented, or at least ameliorated, overshoot and collapse. However, they did not consider why such adaptations did not occur, and in our further development of the BT model here, we do not try to answer the question directly either. (See Ostrom (2000) for a review of knowledge about the evolution of social norms, especially regarding resource management.) Our contribution comprises four new, explicit institutional adaptations, all of which can be viewed as resource conservation "policies", though in practice they would often not be enacted by formal governments, laws and courts; a subsistence requirement in the utility function; and a more general method of analysing the dynamic interaction of these two. We intend these methodological features to be relevant to civilization other than Easter Island, though we have not applied them empirically to any other cases.

The first new feature, a set of four resource conservation policies, comprises a tax, and quotas on total resource harvest, total (harvest) effort, and per capita effort. These are all explicitly defined as additional, collective actions. This departs from previous approaches in the literature, which have been just to make changes in the basic model specifications and/or parameters, and regard these as caused by some unspecified institutional change. Population control policies could be of interest too, but are not considered here.

Our second feature, a requirement in the typical utility function for a subsistence level of harvest, is shown to avoid the implausibility in previous literature that people are assumed to divide their time between harvesting and manufacturing in fixed proportions, no matter how scarce the resource is. ("Manufacturing" here mainly means statue erection and other monument building in the case of Easter Island, but more generally it means anything done with non-harvesting time, as labor is the sole modeled input to manufacturing.) The subsistence requirement is also shown to make the model less stable and more prone to overshoot and complete collapse. This better explains the complete stop in statue manufacturing believed to have happened on Easter Island after about 1500. More generally, it may help explain some of the many sudden collapses of past civilizations (1988), and suggest a further reason why institutional adaptation often does not occur.

The importance of subsistence in economics is widely recognized (see Sharif (1986) for a survey). Its impact on growth patterns has been explored by authors such as Steger (2000), but his and other analyses made more conventional assumptions than ours. Their productive capital stock was man-made rather than natural; instantaneous utility depended only on consumption; the rate of population growth was exogenous; intertemporal rather than instantaneous utility was maximized; and the timescale of interest was much shorter than the many centuries we consider. We hope our analysis will lead to useful insights for contemporary development economics, but the challenge of bridging the paradigms is considerable.

Our third new feature is to investigate how an increased subsistence requirement makes it more difficult, under any of the four conservation policies, to avoid overshoot and collapse. The technique typically used for such investigations would be simulation experiments, with just a few values of the subsistence parameter selected by trial and error. Instead, we use bifurcation analysis to show more rigorously how higher subsistence makes overshoot worse. This is accomplished by categorizing the types of long run development which occur for any combination of the subsistence parameter, and whatever parameter measures the strength of the institution.

Section 2 and the Appendix derive our theoretical results with both a subsistence level of resource, and the four conservation policies. Section 3 uses bifurcation analysis and numerical experiments to examine the effects of subsistence alone (with no institutional adaptation) on stability in general, and manufacturing in particular. Section 4 extends the bifurcation analysis to show that all four policies become less able to avoid overshoot and collapse as the subsistence parameter gets higher. Section 5 concludes.

2 Static and steady state analysis of subsistence, and of resource conservation policies

2.1 Static utility maximization, and the tax policy

Without institutional adaptation, our closed, population-resource model is exactly the BT model, except that our representative consumer is assumed to have an instantaneous utility function of semi-Stone-Geary form,

$$u(h, m) = (h - h_\mu)^\beta m^{1-\beta}; \quad 0 < \beta < 1, \;\; h_\mu > 0 \tag{1}$$

where h and m are respectively individual consumptions of the resource good (that is, h is per capita harvest or nutrition), and of manufactures. Formally, "manufactures" in this model is the output from any time not spent harvesting. This includes more than just artifacts, such as stone statues or tools, that leave durable traces discoverable by archaeologists, and could include ritual and many other non-durable forms of culture. However, it is an acceptable approximation for our purposes to interpret archaeological evidence that the making of durable artifacts ceased at a given time, to imply that all non-harvesting activities ceased then too. The parameter h_μ is a "utility subsistence level", or desired minimum level of resource consumption. This was omitted by BT, who assumed $u = h^\beta m^{(1-\beta)}$. Adding subsistence to the utility function seems appropriate for modeling population collapse in the fairly primitive, resource-based society represented by the Malthusian, nutrition-determined fertility function assumed by BT and recapitulated below. To some extent it also allows the idea of "subsistence stress" which appears frequently in Tainter (1988) to be captured. But as far as we know, minimum required levels of nutrition in both a utility function and a fertility function have not been combined before, and we will discuss in Section 2.3 how they might be interrelated.

Total resource harvest and manufactures consumption are respectively $H = Lh$ and $M = Lm$, where L is the total population at any time (all time arguments are suppressed). Total manufactures M are produced with constant returns to scale using only labor, $M = L_M$, where L_M is the total labor time spent on manufactures. Hence $L_H = L - L_M$ is the total labor time spent on resource harvesting, which we call total effort, using the familiar term from renewable resource economics; and $l_h = L_H/L = 1 - (M/L) = 1 - m$ is per capita effort, or the proportion of an individual's time spent harvesting. For all interior solutions, the manufactured good is taken as the numeraire (i.e. with price = 1) so that the wage rate $w = 1$; and with an individual's available time normalized at 1, total wage income per person is also 1. The (pre-tax) price of the resource in terms of manufactures is p.

One of our four, alternative, institutional departures from BT is that all resource consumption is taxed collectively at an ad valorem rate t, with the revenue being all returned to people as a lump-sum subsidy T. It is convenient to include this in the static analysis here, even though we do not look at its dynamic effect until Section 4. At each moment, the individual consumer myopically maximizes utility (1) subject to the budget constraint

$$(1+t)ph + m = 1 + T. \tag{2}$$

while the condition for overall revenue neutrality (adhered to collectively, but not perceived by any individual) is

$$tph = T. \tag{3}$$

The resulting interior and corner solutions for per capita manufactures and resource consumptions (see Appendix for calculations) are respectively

$$h = \begin{cases} \frac{\beta(1-ph_\mu)}{(1+t-\beta t)p} + h_\mu & \text{if } p < \frac{1}{h_\mu} \\ \frac{1}{p} & \text{otherwise} \end{cases} \tag{4}$$

$$m = \begin{cases} \frac{(1-ph_\mu)(1-\beta)(1+t)}{1+t-\beta t} & \text{if } p < \frac{1}{h_\mu} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

As one would expect, the tax rate t shifts the balance of the interior solutions from resource consumption toward manufactures, but has no effect on the corner solution.

Next, following BT, we assume a Schaefer resource harvest function

$$H = \alpha S L_H, \quad \alpha > 0, \tag{6}$$

where α is the harvestability parameter and S is the resource stock. Given open access, any rent (user cost) of the resource is ignored, and the price of the resource is just the value of labor expended per unit of resource harvested, which from (6) is:

$$p = \frac{wL_H}{H} = \frac{1}{\alpha S}. \tag{7}$$

(4) and (5) then become

$$h = \begin{cases} \frac{\beta(\alpha S-h_\mu)}{1+t-\beta t} + h_\mu & \text{if } \alpha S > h_\mu \\ \alpha S & \text{otherwise;} \end{cases} \tag{8}$$

$$m = \begin{cases} \frac{(1-\frac{h_\mu}{\alpha S})(1-\beta)(1+t)}{1+t-\beta t} & \text{if } \alpha S > h_\mu \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

When analysing quota policies later, we will also need the corresponding formula for total harvest when the tax rate t is zero, which we rearrange slightly to show as a weighted mean:

$$H = Lh = \begin{cases} [\beta\alpha S + (1-\beta)h_\mu]L & \text{if } \alpha S > h_\mu \\ \alpha S L & \text{otherwise.} \end{cases} \tag{10}$$

So if $\alpha S > h_\mu$, the greater the subsistence requirement h_μ, the more that harvest and effort exceed their Brander-Taylor levels.

2.2 Equations of motion, and steady states

As in BT, the resource carrying capacity of the society's land is K, the intrinsic rate of resource growth is r, and the rate of change $\dot{S}$ of the resource stock is natural growth $G(S) = rS(1-S/K)$ minus total harvest $H = Lh$:

$$\dot{S} = rS(1 - S/K) - Lh, \quad r, K > 0 \tag{11}$$

The model is completed by assuming a population growth rate equal to a Malthusian fertility term proportional to per capita resource consumption, $F(H, L) = \phi h, \phi > 0$, minus a base rate of decline. For neatness we write this as σ here rather than $(d - b)$ as in BT.[1] Thus[2]

$$\frac{\dot{L}}{L} = \phi h - \sigma = \phi\left(h - \frac{\sigma}{\phi}\right). \tag{12}$$

Throughout we assume that the parameters satisfy BT's condition (13)

$$K > \frac{\sigma}{\phi\alpha\beta} \tag{13}$$

which guarantees the existence of steady state solution (with a positive population). We denote this and the corresponding steady state resource population as L_∞ and S_∞, respectively. They are found by setting $\dot{S} = \dot{L} = 0$, though it is not necessarily a stable equilibrium, as we discuss below.

The Appendix shows that the steady state resource stock then depends on our new features t, the tax rate, and h_μ, the subsistence level, according to:

$$S_\infty(t, h_\mu) = \begin{cases} \dfrac{\frac{(1+t-\beta t)(\frac{\sigma}{\phi} - h_\mu)}{\beta} + h_\mu}{\alpha} & \text{if } \frac{\sigma}{\phi} > h_\mu \\ \frac{\sigma}{\phi\alpha} & \text{otherwise.} \end{cases} \tag{14}$$

[1]Like BT, we do not distinguish between the effects of nutrition on birth rates (via fecundity and social customs) and on death rates (via disease and perhaps infanticide).

[2]The complete model is thus mathematically similar but not identical to the model of exit-entry in an open access fishery in Clark (1990, p35).

while the steady state population is

$$L_\infty(t, h_\mu) = \frac{r\phi}{\sigma} S_\infty (1 - \frac{S_\infty}{K}) \tag{15}$$

with per capita resource and manufactures consumption being respectively

$$h_\infty = \frac{\sigma}{\phi} \quad \text{and} \quad m_\infty(t, h_\mu) = 1 - \frac{\sigma}{\phi \alpha S_\infty}. \tag{16}$$

With a little manipulation, equation (14) shows that provided $\frac{\sigma}{\phi} > h_\mu$, a positive resource tax will result in a higher steady state resource stock, while a positive subsistence requirement will have the opposite effect. From (14) and (15) the effect of an increased tax rate t on steady state population L is ambiguous: positive if $S < K/2$, but negative if $S > K/2$.

The steady state has another feature which we think has not been noticed in the context of an isolated, population-resource economy but deserves comment. The resource growth function $G(S) = rS(1 - S/K)$ is an upturned parabola, symmetric about the level $S = K/2$ which gives the maximum sustainable harvest. So if $S_\infty < K/2$ – that is, if the steady state entails biological overharvesting (Clark (1990), p27) – then the same or greater harvest, and hence population, can be sustained forever with a larger resource stock, as long as population can be kept in check rather than following a Malthusian dynamic. From (6) and (1), the larger stock means less effort is then needed, and more utility is enjoyed. The institutional adaptations considered in Section 4 will thus be able not only to stabilise development, but also to make people happier in the long run. To illustrate this phenomenon, when $S_\infty < K/2$ we will use the phrase "policy-constrained steady state", denoted by an asterisk, to mean the state with the maximum, larger stock that can sustain the same harvest and population as the steady state. (For simplicity, we will consider it only when the tax rate is zero.) By symmetry, this maximum larger stock is

$$S^* = K - S_\infty > K/2 > S_\infty. \tag{17}$$

To stay in the policy-constrained steady state requires an appropriate check on per capita or total harvest or effort, namely $h \leq \frac{\sigma}{\phi}$, $H \leq L_\infty \frac{\sigma}{\phi}$, $l_H \leq \frac{\sigma}{\phi \alpha S^*}$ or $L_H \leq r(1 - S^*/K)/\alpha$. (The tax will also achieve a similar, but not identical, effect.) Any of these will prevent per capita harvest h rising above σ/ϕ, and thus population temporarily rising above L_∞ by (13).

2.3 Subsistence levels in the utility and fertility functions

From the fertility equation (12), population grows or falls as per capita harvest h is respectively greater or less than σ/ϕ. Since being able to reproduce oneself is arguably a basic need, we will sometimes call σ/ϕ the "fertility subsistence level". How does this relate to the "utility" subsistence requirement h_μ in (1)? To what extent is each level biologically or socially determined?

There are non-trivial questions, and there is scope for further research here. It is tempting to suggest that h_μ is the minimum nutrition needed to stay alive, while σ/ϕ is the nutrition needed for replacement fertility (and is viewed as such by BT, p128), so that $h_\mu < \sigma/\phi$ since one must be alive to have children, but not the other way around.

We are inclined to this interpretation ourselves, but one cannot be sure, as may be seen from the following alternative definitions of a subsistence consumption level in a utility function (from Steger (2000), pp345-6):

1. "...a standard of living that allows for the satisfaction of the minimum (physical and mental) basic needs of life"
2. "...the least-cost requirement for sustaining an individual's dietary needs"
3. "...that amount of consumption which is a necessary prerequisite for sustaining life"

Definitions (2) and (3) just relate to physical nutrition. However, by including "mental" needs, definition (1) suggests that the subsistence level may be subject to social as well as biological influences. Both food and children have social as well biological purposes, so we feel it is worth considering a range of h_μ values in relation to σ/ϕ. However, values significantly above σ/ϕ do seem unlikely and are therefore of less practical interest. The implication of $h_\mu > \sigma/\phi$ is that people choose to put all their labor into harvesting at nutrition levels above those needed to maintain population. So one can guess that a stable population-resource equilibrium may then be impossible to achieve, since a falling population cannot coincide with restraint in harvest effort, and this is indeed what our analysis reveals.[3]

2.4 Quota policies

In Section 4 we analyse the aims and dynamic effects of the tax policy defined in Section 2.1, and three (maximum) quota policies: $H \leq \bar{H}$, a (constant) quota on total harvest; $L_H \leq \bar{L}_H$, a quota on total effort; and $l_H \leq \bar{l}_H$, a quota on per capita effort. These policies are chosen to provide variety compared to the existing literature, and because they roughly correspond to some of the customary restrictions on resource use observed in the anthropological literature.[4]

[3]Where not otherwise specified, by "subsistence level" we will mean h_μ in the utility function, not σ/ϕ in the fertility function.

[4]A quota $h \leq \bar{h}$ on per capita harvest is not considered for the following reason. For simplicity we wish to keep a quota's level constant. During population overshoot, while the other three variables are rising, per capita harvest is falling, so a quota on it would have to be binding from time zero when population is tiny, which seems an implausible institution.

In simple societies without formal governments and taxes, the tax could be thought of as a disincentive to "excessive" resource consumption enacted by social pressures, or possibly some kind of resource-sharing mechanism. The three quotas can likewise be thought of as social norms or taboos on harvest or effort. The information needed to set appropriate quota levels would probably be much more demanding for total rather than per capita levels, but this is a complication which we leave for further work. We give here just the formulae under each quota for total harvest H, which determines the coevolution of S and L in (11) and (12).

The formulae come from (10) by assuming that if any quota requires a per capita harvest less than the subsistence level h_μ, then it is disobeyed, and either h_μ, or the maximum possible level S if this is less than h_μ, is consumed.[5] So with an intended quota of (for example) $\bar{H}$ on total harvest, its formal definition is instead $max(\bar{H}, h_\mu L)$, giving the following:

A quota $\bar{H}$ on total harvest in total harvest:

$$H = \min\{\min[\max(\bar{H}, h_\mu L), (\beta\alpha S + (1-\beta)h_\mu)L], SL\} \tag{18}$$

A quota $\bar{L}_H$ on total effort results in total harvest:

$$H = \min\{\min[\max(\alpha S\bar{L}_H, h_\mu L), (\beta\alpha S + (1-\beta)h_\mu)L], SL\} \tag{19}$$

A quota $\bar{l}_H$ on per capita effort results in total harvest:

$$H = \min\{\min[\max(\alpha SL\bar{l}_H, h_\mu L), (\beta\alpha S + (1-\beta)h_\mu)L], SL\} \tag{20}$$

3 The effect of the subsistence requirement with no institutional adaptation, and the importance of bifurcation

3.1 The effect of the subsistence requirement

Here we are interested only in how the subsistence consumption level h_μ affects the dynamics of the model, and allows for an abrupt disappearance of "manufactures" from a culture. We treat people as uncoordinated individuals, and so ignore the possibility of collective resource

[5]Consideration of policies being disobeyed raises the question of enforcement costs. We investigated the effect of requiring a lump sum cost, in terms of time used for neither harvesting nor manufacturing, to enforce any conservation institution, but found that it made little difference and so added no extra insights. This may well not be true for a more complex and realistic treatment of enforcement costs, which remains an important topic for further research.

conservation, and set the tax rate $t = 0$ or ignore the effect of quotas in any formulae in Section 2.

The effect of the subsistence requirement is fairly easy to understand intuitively. BT's results follow directly from the above by setting $h_\mu = 0$, and thus ignoring all the corner solutions. In BT, no matter how low the resource S falls as a result of harvesting, and thus no matter how negative the population growth rate $\phi\beta\alpha S - \sigma$ (from (8) and (12) with $h_\mu = t = 0$) becomes as a result of famine, people still spend a fixed proportion $m = 1 - \beta$ of their time on manufactures. This means that if resources per person become scarce, population stops growing fairly quickly, putting less pressure on the resource.

However, for people to spend exactly the same proportion of their time on manufactures, no matter how well or poorly fed they or their children are, seems unlikely behavior. Our solutions (8) and (9) with $h_\mu > 0$ allow the more plausible reaction that as the resource stock S falls, labor is reallocated from the manufacturing to the resource sector. Once the stock falls below h_μ/α, and hence the maximum per capita harvest falls below the consumption subsistence level h_μ, manufacturing effort stops altogether. In an important sense, society has then "collapsed", and all human effort is spent on resource harvesting. Even before this, the greater preference for resources caused by $h_\mu > 0$ means that as resources per capita decline, population growth tails off less rapidly, harvesting pressure on the resource is greater, and the overshoot of population is greater, as we shall see. Given the number of fairly complete and rapid collapses of manufacturing observed in the archaeological record (Tainter (1988), Ch. 1), this feature of our model may be relevant to many ancient societies, not just Easter Island which we use as our case study.

As an illustration of the subsistence effect over time, consider Figure 1. Figure 1(A) shows BT's simulation of the resource stock S, population L and total manufactures M on Easter Island over the period 400-1900 A.D., using the parameters in the first column of Table 1. Figure 1(B) shows the development of the same variables, and also total harvesting effort L_H, for the same parameters, except that $h_\mu = 0.015$, which is 60% of the "fertility subsistence", rather than zero. Figure 1(C) shows per capita harvesting effort l_H and manufacturing effort m for both the BT and $h_\mu = 0.015$ cases. Note first, the earlier and much greater population overshoot (to about 16,000 rather than 10,000) which occurs "with subsistence"; second, the much more severe falls in population and resource stocks thereafter; and third, the complete halt in manufacturing which lasts from about 1150 till 1650, by which time the resource (but not yet population) has begun to recover.

Table 1 also shows the steady state values (denoted by an ∞ subscript) of the resource,

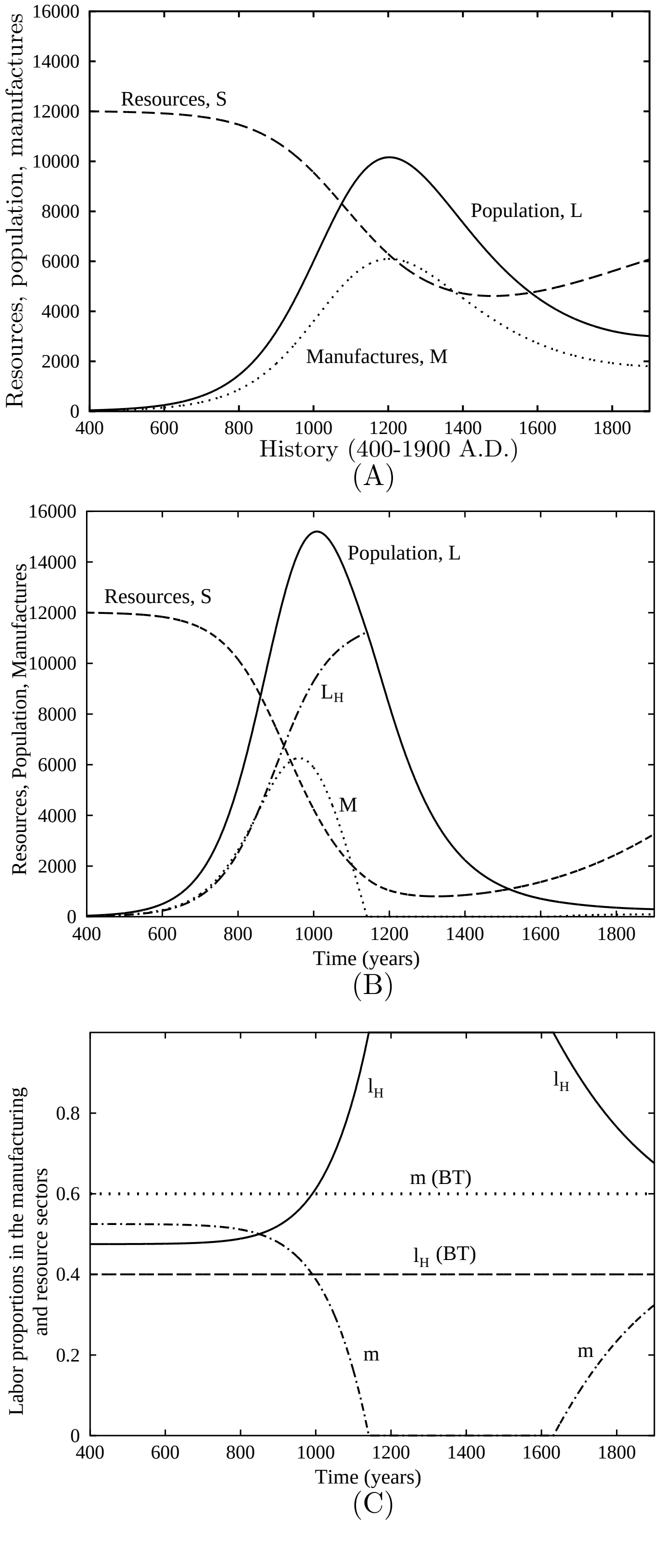


Figure 1: Figure (A) shows the population (solid line), resource stock (dashed line), and manufacturing output (dotted line) on Easter Island for BT's model (where the subsistence requirement $h_\mu = 0$). Figure (B) shows the same, with the addition of total harvesting labor L_H (dot-dash line), for $h_\mu = 0.015$. Figure (C) shows the labor allocations over time as a proportion of the total labor force for the BT model and for $h_\mu = 0.015$, as labelled.

population, per capita and total harvest, and per capita and total effort, for both the BT case of $h_\mu = 0$ and our case of $h_\mu = 0.015$. Since $S_\infty < K/2$, in the latter we include values for the "policy-constrained" steady state described in Section 2.2.

Table 1: Summary of model parameters.

Parameter	Values from BT used in Figures			Alternative values
Fertility, ϕ	4			
Resource taste, β	0.4			
Base rate of population decline, δ	0.1			
Initial population, $L(0)$	40			
Harvestability, α	0.00001			0.000008
Resource intrinsic growth rate, r	0.04			0.035
Carrying capacity, K	12000			11000
Subsistence parameter, and resulting steady state values (* = policy-constrained)				
Utility subsistence level, h_μ	0	0.015		0.017
Equilibrium resource stock, S_∞	6250	4000	8000	4625
Equilibrium population level, L_∞	4792	4267	4267	3753
Equilibrium total harvest, H_∞	119.8	106.7	106.7*	N/A
Equilibrium per capita harvest, h_∞	0.025	0.025	0.025	N/A
Equilibrium total effort , L_{H_∞}	1917	2667	1333*	N/A
Equilibrium *per capita* effort , l_{H_∞}	0.4	0.625	0.3125*	N/A

3.2 Long term dynamics, and bifurcation analysis

Subsistence also causes differences not just initially, but also in long-term stability behavior. Figure 2 shows just population for the Figure 1(A) and 1(B) simulations with a compressed timescale, which extends out as a "prediction" of what would have happened up to 4900 A.D. with no outside intervention. Note how much more rapidly (if "rapidly" is the right word for changes happening over several centuries) the damped oscillations of population converge when there is no subsistence. Indeed, suppose the initial population of Polynesian adventurers had reached Easter Island at least three thousand years earlier, and the BT model had been correct. Then Captain Cook would have found a very stable population and culture, which might have become a historical example of sustainable resource use much admired by modern environmentalists! But under the subsistence model, evidence of quite separate eras of monument-building (and maybe no current building) might have been found. So a subsistence requirement causes not just greater initial overshoot, but a much slower damping of the ecological-economic system.

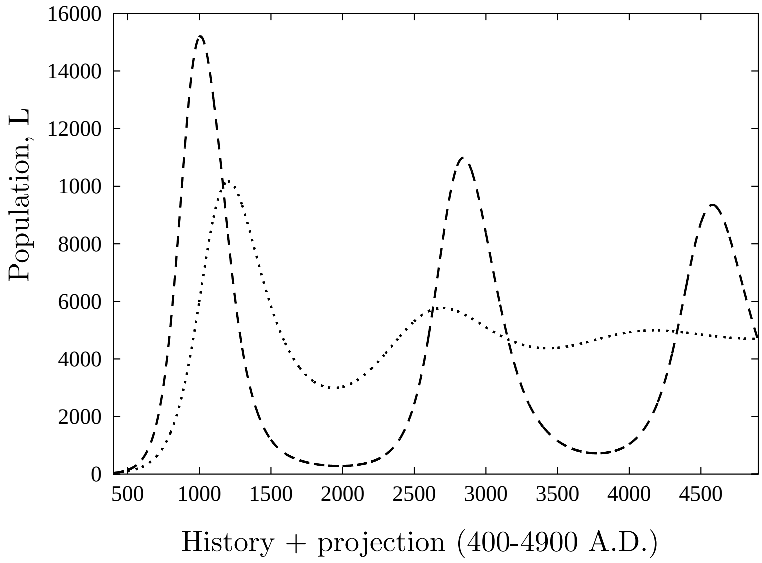
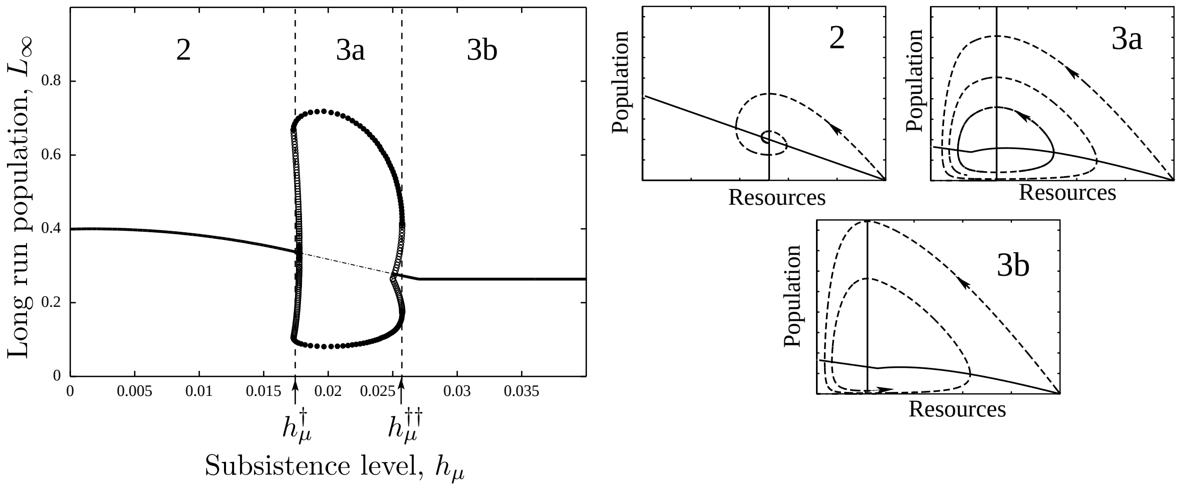


Figure 2: Population for the BT model (dotted line) and subsistence model (dashed line).

Furthermore, when the subsistence level h_μ rises above some threshold $h_\mu^\dagger$, which depends in a complex way on the parameters, the model behavior fundamentally changes, in a way that could matter for many applications of BT-like models. For $h_\mu < h_\mu^\dagger$, the steady state is a stable equilibrium in the (S, L) phase space, toward which the system will converge either monotonically or through a series of damped oscillations. For $h_\mu > h_\mu^\dagger$, the steady state is unstable and the system will converge to a limit cycle circling around the steady state. The transition between the two behaviors is known as a *Hopf bifurcation* (Kuznetsov, 1995).

Figure 3(A), computed using the pseudo arclength continuation technique in the software package Auto (Doedel, 1981), shows this behavior shift in detail. It plots the steady state population L_∞ in (15) as h_μ increases (shown as a continuous fine line), the bifurcation point $h_\mu^\dagger$ (= 0.01775 for the BT parameter set), and the upper and lower values of the limit cycle which occur for $h_\mu^\dagger < h_\mu < h_\mu^{\dagger\dagger}$ (circles).

The regions on the Figure are numbered 2, 3a and 3b to match corresponding regions in other Figures below. Region 2 is where the steady state is stable. As h_μ rises through $h_\mu^\dagger$, the real parts of the two eigenvalues of the Jacobian of the system change from negative to positive. This change in the real parts of the eigenvalues from negative to positive weakens the natural damping in the model, leading to more extreme fluctuations. Region 3a is where



(A) (B)

Figure 3: Figure (A) shows a bifurcation diagram showing fundamental change in long term behavior as h_μ is varied. Figure (B) shows the corresponding phase plane diagrams for each of the regions is Figure (A). In each figure the vertical solid line shows the population nullcline (points where $\dot{L} = 0$), hereafter PN), and the sloping solid line shows the resource nullcline, (RN). The dashed line shows a typical trajectory. The subsistence parameter changes the topology of the model in two ways: by shifting the PN to the left and by causing the RN to become curved. The equilibrium point where the PN and RN cross is stable if the slope of the RN is negative as in 2. The combination of leftward movement of the PN with the curvature of the RN gives rise to situations in which the slope of the PN is positive at the intesection point as in 3a. In this case, the equilibrium point is unstable and the limit cycle emerges. Increasing h_μ still further shifts the PN further left into the region where the RN becomes linear with a negative slope where again the equilibrium is stable as in 3b. However, the topological feature of the curved RN remains, generating severe overshoot and collapse dynamics.

the steady state is unstable and the model converges to a limit cycle. Because the value of $h_\mu = 0.015$ chosen for Figures 1(B), 1(B), and 2 is rather close to $h_\mu^\dagger$, the real parts of the system eigenvalues are then quite close to zero, though still negative. So although the steady state is still stable, convergence is much slower than with $h_\mu = 0$, as shown by Figure 2.

The transition from region 3a to region 3b is marked by a saddle-node of periodics bifurcation at $h_\mu = h_\mu^{\dagger\dagger}$ (= 0.02573), where the limit cycles become unstable. For $h_\mu > h_\mu^{\dagger\dagger}$, the limit cycle behavior ceases and the model again converges (though very slowly) to a steady state. Figure 4(B) illustrates all these behaviors with phase-plane diagrams, as explained in the figure caption. However, though the division between regions 3a and 3b is mathematically interesting, it is not very meaningful because it occurs where $h_\mu > \sigma/\phi$, which is less likely to occur in practice for reasons discussed in Section 2.3. So from now on we merge regions 3a and 3b into a single region 3, and what matters is that as a larger subsistence parameter takes the model further from region 2 into region 3, there will be increasingly severe overshoot and collapse behavior.

Before going on, note that our subsistence model can give a better fit than the BT model to what is known of Easter Island history, by using the parameters in the final column of Table 1. Some of these are the same as BT's, while others are "tweaked" but still plausible. The resulting simulation has the same peak population of about 10,000, but in about 1320 A.D. rather than BT's 1230, closer to the target date of 1400. It has a complete cessation of manufacturing in about 1500, in line with the archaeological evidence, whereas manufacturing in the BT model is still at about half its peak then. Finally, our population in 1774 is about 2500, closer to the 2000 estimated by Captain Cook's party than the BT figure of about 3200.

4 Analysis of institutional adaptations: How subsistence makes it harder to avoid overshoot

4.1 Types of institutional adaptation, and aim in studying them

Before giving our analysis of four variants of explicit institutional adaptation, we state briefly what other variants of adaptation have already been considered in the literature, and then in detail the motivation for studying this topic. In Reuveny and Decker (2000), "population management reform" caused a parametric change in the fertility function. In Dalton and Coats (2000), "changes reflecting the nature of property rights over the resource" caused a parametric change in the degree to which the rate of resource stock growth leads to higher effort. In Erickson and Gowdy (2000), where accumulated (but depreciating) human-made capital substitutes for nutrition in raising fertility, "caring for the future" and "an ethic of zero population growth" eliminated capital depreciation, and halved the effect of human-made capital on fertility. By contrast, our institutional adaptations - the ad valorem, revenue-neutral resource tax in (2) and (3), at a time-varying rate t described below, and constant quotas $\bar{H}$ on total harvest, $\bar{L}_H$ on total (harvest) effort or $\bar{l}_H$ on per capita effort, as defined by (18)-(20) respectively - are explicit policy mechanisms to conserve the resource stock, added as extra incentives or constraints on the model.

The common motivation is to see what kind of institutional adaptations make overshoot and collapse of population less or more severe. But how should a society's interest in avoiding overshoot and collapse be defined? Koopmans (1977) emphasized that when population varies, as here, agreeing on a dynamic goal to determine society's choices over the course of history raises difficult questions. Is society's wellbeing at each time z seen as population $L(z)$, per capita utility $u(z)$, or total utility $L(z)u(z)$? And is society's goal at each time then to maximize current wellbeing, its long-run steady-state level, or its discounted present value? In a model with endogenous population growth over centuries, BT chose the maximization of current per capita utility as the goal, but in Steger's (2000) model with subsistence consumption and exogenous population growth over (presumably) decades, the goal was to maximize the present value of total utility.

In practice, society's dynamic goal will of course adapt over time under a complex set of influences: institutional adaptation is a gradual process. However, in common with existing literature, we do not explicitly analyse this process of adaptation, which remains as a considerable challenge for further work. In comparing our institutional adaptations, we adopt the following, we hope plausible, view of society's dynamic goal, even though it is by no means fully explained, and not a formal maximization. We assume society would like to know the minimum necessary action, for a given type of policy, to avoid overshoot. We look for the minimum, because stronger action tends to reduce steady state population, which is considered undesirable. We define "overshoot" to have been avoided if population peaks at no more than 15% above its long run steady state level. This was chosen because we found that any decline of less than about 15% from a peak takes place over a very long time, and therefore can be considered mild; and avoiding decline completely requires draconian policies which greatly reduce steady state population. And as stated earlier, we wish to determine whether an increased subsistence parameter h_μ makes it harder to avoid overshoot.

4.2 An ad valorem resource tax

Given the problem of overshoot shown in Section 3, which all our four resource conservation policies aim to solve, any policy needs to be "switched on" not until well after the start of history, but before the resource stock S gets too small and the population L gets too large. With quotas, this switching on is fairly automatic. With the tax, we could simply switch it on from some date onwards, but instead we choose an automatic mechanism to phase it in, by letting the tax rate depend inversely on the per capita resource stock S/L at any time according to the following formula:

$$t = t_{max}[1 - s^b/(a^b + s^b)] \tag{21}$$

where $s \equiv S/L$. The rationale behind (21) is that as the *per capita* resource stock decreases to some threshold, people start to take collective action to preserve it. Although the precise functional form of (21) is arbitrary, it is also very general, and any similar switching-on function would produce similar qualitative behavior. The parameters a and b can be chosen to produce a wide range of behaviors from the tax switching on immediately once s falls to the threshold a and then remaining at t_{max} (equivalent to a constant tax), to a case where the tax is phased in gradually over time. For our standard Easter Island case, we used $a = 5$ and $b = 3$, which makes t/t_{max} "turn on" from about 0.2 when $s = 8$, to about 0.8 when s falls to 3. Since the equilibrium value of s is 1.3, this parameter choice represents a relatively proactive population.

As an illustration, we show in Figure 4 how this tax affects the development of the resource stock, population and manufacturing levels over 400-1900 A.D. for the standard BT parameters for Easter Island. The subsistence requirement h_μ is 0.015, so Figure 4(A) can be compared with Figure 2(A) showing development with the same parameters but no tax. The contrast is obvious: the tax avoids overshoot, with population peaking in about 1450 at only 3077, less than 6% above its steady state level of $L_\infty = 2912$, though the latter is 32% lower than with no tax. Moreover, the resource stock converges to a steady state level of 9762, which is above the policy-constrained level of 8000 shown in Table 1.

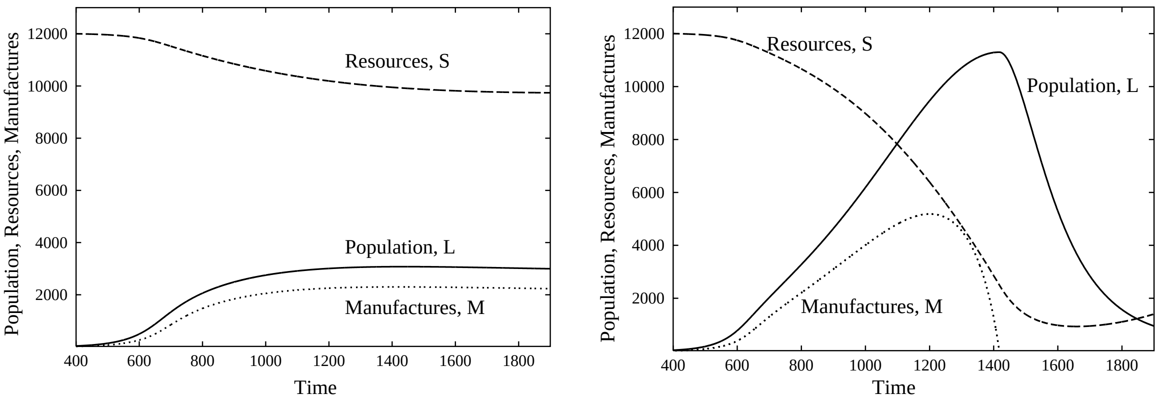


Figure 4: Figure (A) shows the population (solid line), resource stock (dashed line), and manufacturing output (dotted line) for the subsistence model with the parameter set in column two of Table 1, and a maximum tax strength $t_{max} = 5$. Figure (B) shows the same variables for the same model, but with subsistence $h_\mu = 0.025$ instead of 0.015.

However, if t_{max} is held constant but the subsistence requirement increased to $h_\mu = 0.025$, development follows a very different path, as shown by Figure 4(B). Overshoot and collapse still occur, and in fact if the time axis is greatly extended, we would see that the system is stuck in a limit cycle. This supports, but in no way proves, our earlier suggestion that increased subsistence generally makes it much harder for institutional adaptation to prevent overshoot and collapse. However, we can do considerably more, by using bifurcation analysis to show exact ranges of subsistence values h_μ in which our particular institutional adaptation, the ad valorem tax t, can or cannot prevent overshoot. This is done by exploiting the fact that the point at which the Hopf bifurcation occurs is a function of both h_μ and t_{max}. Using sophisticated bifurcation software, we computed the value $h_\mu^\dagger(t_{max})$ of the Hopf bifurcation point, first shown in Figure 4 with no tax, for any value of the maximum tax. The result is the solid line in Figure 5, which we call a "policy analysis diagram" because it divides the (h_μ, t_{max}) parameter space into regions where the tax policy has qualitatively different long run effects. At any point to the right of this bifurcation line (i.e. in region 3), the economy follows either a limit cycle (and hence no steady state), or (for even higher values of h_μ) very slow convergence to the steady state.[6] The dotted line to the left of the bifurcation line divides the region where moderate convergence to a steady state occurs into region 1 where (more than 15%) overshoot does not occur, and region 2 where it does. The parameter combinations (0.015,5) for Figure 4(A) and (0.015,5) for Figure 4(B) fall in regions 1 and 3 of Figure 5 respectively.

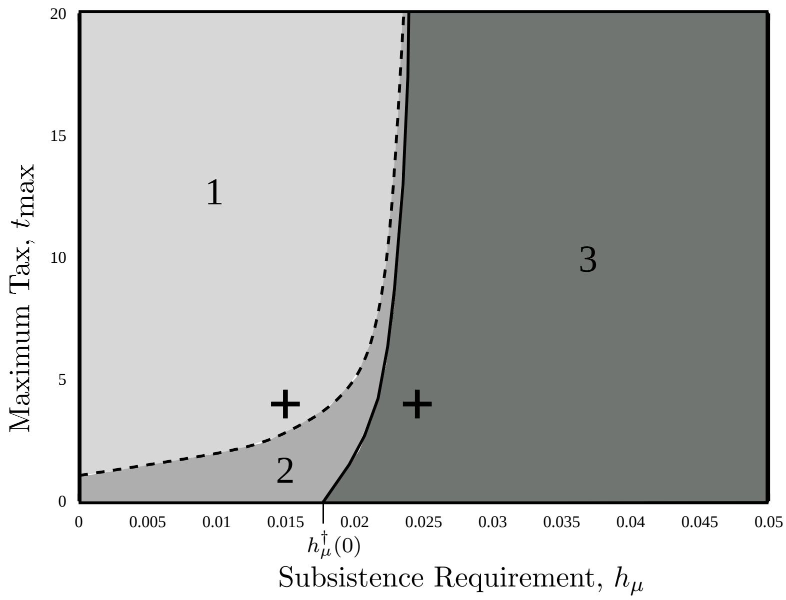


Figure 5: Regions in $h_\mu - t_{max}$ parameter space with qualitatively different behavior. In region 1, institutional response will prevent overshoot and collapse. In region 2, the system will experience relatively mild overshoot and collapse and eventually converge to a steady state. In region 3, the system will experience increasingly severe overshoot and collapse as h_μ increases. The system will either oscillate perpetually or converge to the steady state relatively slowly. The two crosses mark the parameter combinations used for Figures 4(A) and (B).

The positive slope of both lines shows the destabilizing effect of an increased subsistence requirement h_μ on the system, as follows:

(a) for $0 < h_\mu < h_\mu^\dagger(0)$, overshoot but not perpetual oscillation occurs with no tax. A tax (i.e. upward movement from region 2 to region 1) can prevent overshoot, but the tax strength needed to do this (as measured by t_{max}) rises as h rises;

(b) for $h_\mu^\dagger(0) < h_\mu < 0.025$ (the vertical asymptote of the solid line), perpetual oscillation or overshoot with very slow convergence occurs with no tax, but a strong enough tax (approaching infinity as h_μ approaches 0.025) can prevent overshoot;

(c) for $h_\mu > 0.025$, perpetual oscillation or overshoot and very slow convergence occurs, no

[6] The reasons for the latter behaviour were given in the discussion of Figure 3.

matter how strong the tax.

4.3 A quota on total harvest

The instantaneous effect of a quota $\bar{H}$ (constant over time) on total harvest H was given by (18). To make a general analysis of the dynamic effects of this policy, we go straight to the policy analysis diagram for this policy, still using the same BT parameters from Table 1. In Figure 6, the space spanned by the subsistence requirement h_μ and the quota $\bar{H}$ falls into the same three areas as Figure 5, but with very different shapes.

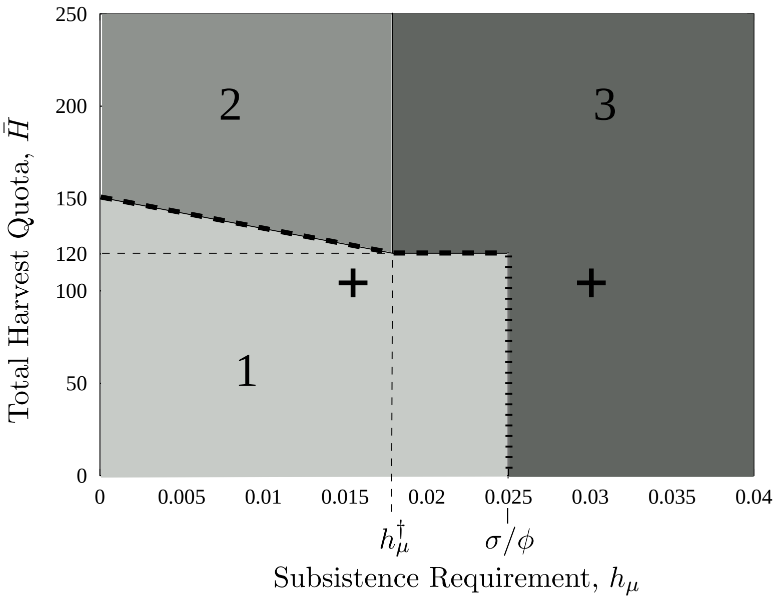


Figure 6: Regions in $h_\mu - \bar{H}$ parameter space with qualitatively different behavior. In region 1, institutional response will prevent overshoot and collapse. In region 2, the system will experience relatively mild overshoot and collapse and eventually converge to a steady state. In region 3, the system will experience increasingly severe overshoot and collapse as h_μ increases. The system will either oscillate perpetually or converge to the steady state relatively slowly. The two crosses mark the parameter combinations used for Figures 7(A) and (B).

Owing to the three inequality constraints (the quota itself, the rule for disobeying the quota, and the corner effect of the subsistence requirement) built into Figure 6, the explanation of these shapes is quite complex and only moderately insightful. So it is omitted, but is available from the authors. Here we just note that a rising subsistence parameter h_μ makes overshoot harder to avoid, as follows. (Note that the policy-free case is beyond the top of Figure 6 – a quota $\bar{H}$ so high that it has no effect – rather than at the bottom of Figure 5, a zero tax.)

(a) For $0 < h_\mu < h_\mu^\dagger$, the harvest quota can avoid overshoot, but it has to become tighter (i.e. lower) as h_μ increases, giving the downward sloping line between regions 1 and 2.

(b) For $h_\mu^\dagger < h_\mu < \sigma/\phi$, a quota of $\bar{H} < 120$ will change the model's long run behavior from a limit cycle to no overshoot.

(c) For $\sigma/\phi < h_\mu$, no quota can prevent overshoot, mainly because it will be disobeyed if it implies a per capita harvest below h_μ.

As an illustration, Figure 7 shows the historical development of total resource stock, population and manufacturing for the parameter combinations $(h_\mu, \bar{H}) = (0.015,106.7)$ and (0.03,106.7), which fall respectively in regions 1 and 3 of Figure 6. Figure 7 is thus comparable to the development histories shown in Figure 4 for the resource tax. In addition we have shown by the bold solid line when the quota actually binds. In Figure (A), the quota binds forever from about 700 A.D onwards. The resulting steady state, which is reached with no overshoot at all, has a resource stock equal to the policy-constrained value in (17), of $S^* = 8000$. It has a population equal to the no-adaptation value in (15) of $L_\infty = 4267$, in contrast to the tax case in Figure 4(A) where steady-state population is nearly a third below L_∞. However, at twice the subsistence level, the quota binds only for a relatively short period of time during overshoot and collapse in Figure (B), after which it is disobeyed.

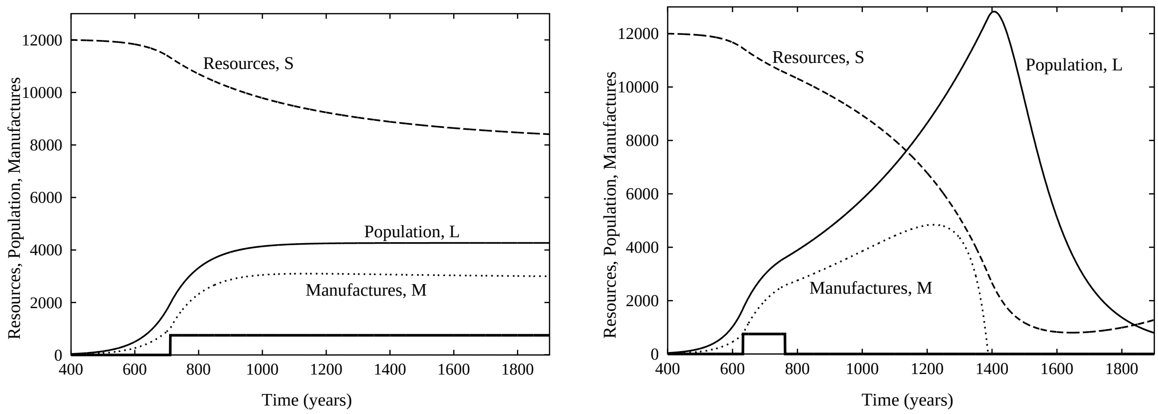


Figure 7: Figure (A) shows the population (dashed line), resource stock (dotted line), and manufacturing output (solid line) for the subsistence model with the parameter set in column two of Table 1, and a total harvest quota $\bar{H} = 106.7$. The bold solid line shows when the quota binds. Figure (B) shows the same variables for the same model, but with subsistence $h_\mu = 0.03$ instead of 0.015.

4.4 Quotas on total effort and per capita effort

The effect of a quota on total (harvest) effort, as defined in (19), is shown in the corresponding policy analysis diagram, Figure 8. It is similar to Figure 6 in that the quota has an "on-off" nature. Further explanations of the region boundaries are available from the authors. Illustrations of dynamic behavior for parameter combinations in regions 1, 2 and 3 are not radically different from the case of a total harvest quota, and so are not given.

The policy of a quota on *per capita* (harvest) effort, $l_H \leq \bar{l}_H$ as defined in (20), does not require bifurcation analysis to see its effect (though the following argument is far from a rigorous

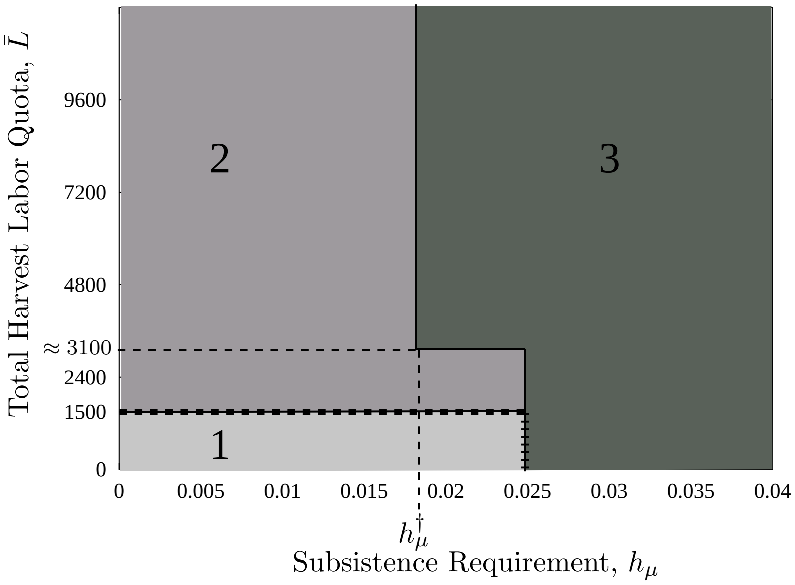


Figure 8: Regions in $h_\mu - \bar{L}$ parameter space with qualitatively different behavior. In region 1, institutional response will prevent overshoot and collapse. In region 2, the system will experience relatively mild overshoot and collapse and eventually converge to a steady state. In region 3, the system will experience increasingly severe overshoot and collapse as h_μ increases. The system will either oscillate perpetually or converge to the steady state relatively slowly.

proof). First, if $h_\mu = 0$, recall that for BT parameters, $l = 0.4$ always. So in the BT model, a quota $\bar{l}_H > 0.4$ has no effect, and thus will not prevent overshoot. If $h_\mu > 0$, it turns out that a quota $\bar{l}_H > 0.4$ will not prevent overshoot either, for the crucial reason that once the quota requires a per capita harvest $h < h_\mu$, it is disobeyed. So like the case with no institutional adaptation, $h_\mu > 0$ simply makes overshoot more severe. For $\bar{l}_H < 0.4$, control of harvest effort is so severe from the start that the subsistence requirement turns out never to bite. Such a society is so disciplined that individuals put the common good before individual utility from the start. Thus, a broad pattern emerges that quotas prevent overshoot only when they are draconian, and then their effect is rather obvious.

5 Conclusions

We have made three additions to Brander and Taylor's (1998) model of an isolated, myopic, Malthusian human society which depends solely on a renewable natural resource with a finite carrying capacity, where population is inherently capable of overshoot and collapse. First, adding a minimum subsistence level of resource consumption allows the proportion of time spent resource harvesting to vary with resource scarcity, including the corner solution where all time is spent harvesting. This is consistent with the sudden and complete disappearance of "manufacturing" (non-harvesting activities such as monument-building which produce durable remains) which has often been observed in the historical record, and which happened when the famous, giant stone statues were no longer made on Easter Island after about 1500 A.D. More generally, subsistence destabilizes development, because it results in a less effective response to declining resources: people try to maintain their resource consumption, rather than allowing population to decline faster, so overshoot and collapse becomes more severe. We suggested that the subsistence level should be less than the implicit subsistence level required for population stability, but this may repay further investigation.

Second, we explored four forms of institutional adaptation to prevent overshoot and collapse: a time-varying, ad valorem tax on resource use, and constant quotas on total harvest, total (harvest) effort, or per capita effort. Each of these resource conservation policies is intended to avoid a population overshoot of more than 15% above a steady state value, and was defined as an explicit collective mechanism, rather than as a change in basic model dynamics or parameters. We chose a rule that all quotas were disobeyed once consumption is below the subsistence level. This has a big effect on dynamics for high subsistence levels, and identifies the cost of informing and enforcing various quota levels as a significant issue for further work. Broadly, quotas prevent overshoot only when they are draconian, and then their effect is rather obvious.

Our third innovation was to analyse the long run dynamic effects of any combination of subsistence level and conservation policy, by using bifurcation analysis combined with numerical experiments. The resulting policy analysis diagrams show how a higher subsistence requirement fundamentally destabilizes development. Either a stronger policy is needed to prevent overshoot, or for the highest level of subsistence requirement, no policy can prevent overshoot, and a limit cycle of perpetual overshoot and collapse is often the result. However, a bonus stemming from the logistic resource growth function assumed is that resource conservation policies can increase steady state utility without reducing steady state population, by preventing biological overharvesting.

Determining to what extent such theoretical predictions can explain why known past civilizations experienced just one, or many, cycles of overshoot and severe or complete collapse, is a large task remaining for further work. Since cycle times can have an order of magnitude of a thousand years, distinguishing an oscillation converging to a steady state from one converging to a limit cycle would not be easy. Another large question also remains unanswered: why or why not have various institutional adaptations been adopted? How do societies evolve institutions which set some long term goal, instead of instantaneous utility maximization, for harvesting decisions? Though we have not directly tackled this, our demonstration that a subsistence requirement makes long-term management of renewable resources more difficult suggests that ignoring subsistence may lead to misleading analysis of institutional evolution, and ineffective policy proposals. Many other model developments - time-varying quotas, population controls, accumulation of monuments, irreversibility of resource exhaustion, and so on - are possible and interesting, but we suggest that including subsistence, and using bifurcation techniques of dynamic analysis, are innovations worth repeating for this type of ecological-economic modeling.

A Appendix: Instantaneous Utility Maximisation, and Steady States

A.1 Consumer optimization

To derive results (4) for h and (5) for m from the constrained optimization problem defined by (1), (2) and (3), define the Lagrangian

$$\mathcal{L} \equiv (h - h_\mu)^\beta m^{1-\beta} + \lambda[1 + T - (1 + t)ph - m] \tag{22}$$

First order conditions for an interior solution are:

$$\frac{\partial \mathcal{L}}{\partial h} = \frac{\beta u}{h - h_\mu} - \lambda(1 + t)p = 0 \quad \Rightarrow \quad \frac{u}{\lambda} = \frac{(1 + t)p(h - h_\mu)}{\beta} \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial m} = \frac{(1 - \beta)u}{m} - \lambda = 0 \quad \Rightarrow \quad \frac{u}{\lambda} = \frac{m}{(1 - \beta)} \tag{24}$$

$$(2) \text{ and } (3) \quad \Rightarrow \quad m = 1 - ph \tag{25}$$

$$(24) \text{ and } (25) \quad \Rightarrow \quad \frac{u}{\lambda} = \frac{1 - ph}{1 - \beta} \tag{26}$$

$$(23) \text{ and } (26) \quad \Rightarrow \quad \frac{(1 + t)p(h - h_\mu)}{\beta} = \frac{1 - ph}{1 - \beta} \tag{27}$$

which after routine algebra gives (4) and then (5). Since a negative amount of manufactures is physically impossible, the condition for this interior solution to apply is

$$p < \frac{1}{h_\mu} \tag{28}$$

If (28) does not hold, then a corner solution (in which the tax rate plays no part) applies instead:

$$h = \frac{1}{p}, \quad m = 0. \tag{29}$$

A.2 Steady state

To derive the steady state for S in (14), L in (15), h and m in (16) corresponding to the first value of $S_\infty(t, h_\mu)$ in (14), setting $\dot{L} = 0$ in (12) gives h immediately. Using (8) then gives

$$\phi \left[\frac{\beta(\alpha S - h_\mu)}{1 + t - \beta t} + h_\mu \right] = \delta \tag{30}$$

$$\Rightarrow \ \alpha S - h_\mu = \frac{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)}{\beta} \tag{31}$$

from which (14) and (15) follow by straightforward algebra, including setting $\dot{S} = 0$ in (11) (with no check if the resulting steady state is stable). Next,

$$(14) \ \ \Rightarrow \ \ 1 - \frac{h_\mu}{\alpha S} = 1 - \frac{h_\mu}{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)/\beta + h_\mu} \tag{32}$$

$$= \frac{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)/\beta}{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)/\beta + h_\mu} \tag{33}$$

which with (11) gives m in (16) as required.

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