Sustainability Policy and Environmental Policy

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Abstract. A representative agent economy with a resource stock, polluting emissions, productive, abatement and foreign capital, trade, and technical progress, is used to contrast environmental policy, which internalises amenity and productivity values, with sustainability policy, which achieves some form of intergenerational equity. Environmental policy comprises a tax on emissions and a subsidy on the resource stock equal to the respective externalised costs or benefits. Sustainability policy comprises a capital and/or consumption tax to change the effective utility discount rate. Environmental policy can reduce the strength of sustainability policy needed. More specialised results are derived in a closed economy with a non-renewable resource and no technical progress, and in a small open economy with few environmental effects.

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1. Introduction

In the voluminous government literature on sustainable development since the Brundtland report (WCED 1987) popularised the idea, it is often hard to distinguish sustainability policy from environmental policy. A document on a country's approach to sustainable development may start with general statements about sustainability as safeguarding the well-being of future generations, but then continue with little more than a description of environmental policies. Detailed policies are spelt out on air and water pollution, solid waste, traffic congestion, habitat and biodiversity protection, etcetera, perhaps with special emphasis on the enduring nature of some forms of environmental damage; but nothing is said about other policies that may be necessary or desirable to sustain well-being.

If governments truly believe there are fairly imminent limits to the substitutability of human-made capital and knowledge for environmental resources, then this implicitly "strong" approach to sustainability, which treats environmental protection as the essence of sustainability policy, would be logical. But by the very limits of their environmental policies, most governments reveal that they do not much believe in such limits to substitutability. (We express no view here on whether, and if so where, limits to substitutability do actually exist. At the macroeconomic level, this is a hugely difficult empirical question which has so far eluded any answer which commands consensus. This is why we write about what governments believe, rather than what is true.) Most governments implicitly place finite rather than infinite values on marginal declines in environmental resources. So it seems relevant at least to current policy debate to adopt here the "weak" approach to sustainability, and assume capital-resource substitutability at the margin.

Given this, we confirm here the obvious intuition that if it is needed at all, sustainability policy should include non-environmental aspects of providing more for future generations, such as encouraging more saving and hence capital investment to substitute for some degree of future environmental resource depletion. Sustainability and environmental policies will thus be at least partially distinct. The aim of this paper is to clarify these distinctions, using a conventional theoretical framework of a dynamic, optimising economy with identical agents, represented by one agent making decisions in continuous time.

To do this, we must define the two types of policy more precisely. We assume that each agent would like to optimise society's development by maximising some well-defined measure of intertemporal welfare, based on individual preferences. However, competitive markets alone will not maximise welfare, because each agent treats various environmental costs that affect welfare as external to their private optimising decisions. *Environmental policy* is then the time path of all incentives, such as emission taxes and resource conservation subsidies, with which the government can intervene in decentralised markets to internalise these externalities, and thus achieve the optimal path of development. We do not enquire into, and our theoretical model will not distinguish, when and why tradeable emission permits or (non-tradeable) emission standards might be preferable to taxes and subsidies, or what are the efficiency gains of taxes or tradeable permits over standards.

By contrast, *sustainability policy* is the time path of incentives which together persuade agents in a decentralised economy to achieve a collectively-desired "sustainability" goal. Such a goal is most generally defined as any departure, from maximising a social welfare function that is based solely on individual agents' own time preferences, in a way that could be said to improve intergenerational equity. Within this, we are especially interested in departures that focus on the path of per capita utility over time, for example by requiring that utility remain forever constant, non-declining, or sustainable, since we feel these are the most natural formalisation of "safeguarding the well-being of future generations". If greater precision and simplicity in analytic work is called for, we use forever constant utility as a sustainability goal.

The conventional (neoclassical) literature on sustainability has mostly focused on defining and justifying it,[1] or on measuring it,[2] rather than on identifying policies to achieve it, which is our focus. However, there is also plenty of literature about environmental and/or general intergenerational policies in a dynamic economy, which unless otherwise stated uses an overlapping generations rather than a representative agent (RA) demographic format for society. For our purposes, it is relevant to divide this literature into four parts. First, papers like John and Pecchenino (1994) and Smulders (2000, in RA format) provided analysis only from the viewpoint of a social planner. Thus no policy instruments as such were considered; though John and Pecchenino set the "golden rule" goal of maximising steady state utility, and Smulders noted whether or not consumption (and thus utility) rose or

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[1] For examples, see Howarth (1992), Dasgupta (1994), or Chichilnisky (1996) and Asheim, Buchholz and Tungodden (2001). Solow (1993, p171) made the target of a sustainability policy clear, with his claim that "...we owe to the future a volume of investment that will compensate for this year's withdrawal from the inherited stock". However, he gave no analysis of what policy instruments can achieve this volume of investment if market forces do not.

[2] For examples, see Pearce and Atkinson (1993), Asheim (1997), Aronsson et al (1997), Atkinson et al (1997), Brekke (1997) and Weitzman (1997).

fell on a steady state path. Second are papers which consider a dynamic instrument of environmental policy to internalise externalities, but either no explicit sustainability goal (as in Bovenberg and Smulders 1995, Mohtadi 1996, both in RA format, and Jouvet et al 2000), or no policy to achieve this goal (John et al 1995, which again adopted a golden rule goal for the social planner). Third are papers like Howarth and Norgaard (1990), Mourmouras (1993) and Krautkraemer and Batina (1999), which analysed some kind of sustainability goal, but had no conventional externalities, and hence no environmental policy.

Finally there are papers considering both environmental policy and sustainability policy. Howarth and Norgaard (1992), Marini and Scaramozzino (1995), Howarth (1998), Bovenberg and Heijdra (1998) and Gerlagh and Keyzer (2001) all used an overlapping generations format; this allows the use of intergenerational transfers as a key instrument of sustainability policy, in pursuit of goals such as maximising the undiscounted sum of generations' lifetime utilities. Such transfers will not be available here because of our RA format. Becker's (1982, in RA format) instrument to achieve maximin (hence constant) utility is direct manipulation of the interest rate path, which we exclude: it would be difficult to sustain in a small open economy, and in any case is not easy to reconcile with the use of interest rates for macroeconomic stabilisation. The analysis in Pezzey (1992) foreshadowed that here, but was much simpler and more restricted.

What is new about our treatment, in comparison to this existing literature on both environmental policy and sustainability policy, is the combination of the following features. Pollution, resource depletion, physical capital accumulation, trade and (exogenous) technical progress are all included (but not education and knowledge accumulation). Compared to much literature, our functional forms are more general. The focus is on transitional rather than steady state paths. It is shown how applying environmental policy may reduce the required strength of sustainability policy. Finally, of many policy instruments considered, the natural ones to use for sustainability policy in an RA format, at least in theory, turn out to be a capital subsidy or falling consumption tax rate, which together encourage saving now and consuming later. Our focus on encouraging saving as a sustainability policy is supported by empirical measurement studies following the example of Pearce and Atkinson (1993), but has so far received little attention in the policy literature.

Our framework also allows two more specialised topics to be investigated. One is that resource incentives cannot prevent unsustainability in the form of asymptotically declining utility, in a closed economy with constant discounting, no resource discovery or renewal, and no technical progress. The other is that sustainability policy has no effect at all on resource management and domestic production and investment, in a small open economy with few environmental effects.

The paper is organised as follows. Section 2 describes the most general economy to be considered, along with reasons why full generality cannot be achieved, and derives the environmental policies needed to internalise all externalities. Section 3 discusses but does not formally analyse a rationale for sustainability policy, and then analyses this policy in the same economy, and its general interaction with environmental policy. Section 4 uses specific functional forms to show a precise effect of environmental policy on sustainability policy. Section 5 considers the two more specialised topics just mentioned, and Section 6 concludes.

2. Environmental policy in a fairly general economy

2.1 The economy

A fully general representative-agent, continuous-time, deterministic economy would be similar to that in Asheim and Weitzman (forthcoming). At any time $t \geq 0$, there is a "consumption" vector $\mathbf{C}(t)$ of everything, including environmental amenities, that influences the representative agent's instantaneous utility $U(\mathbf{C}(t))$. Likewise, there is a vector $\mathbf{K}(t)$ of stocks of built, knowledge and human capital, and of all environmental resources. The relationship between $\mathbf{C}$ and $\mathbf{K}$ can be given by a convex production possibilities set $\Pi\{.\}$:

$$[\mathbf{C}(t),\dot{\mathbf{K}}(t)] \in \Pi\{\mathbf{K}(t),t\} \qquad [1]$$

in which we have included the possibility of exogenous shifts in production possibilities, as shown by the dependence of $\Pi\{.\}$ on t as well as $\mathbf{K}$.

However, the form of [1] is *too* general for our purposes. It hides specific features that make environmental resources important to policymakers. In [1] one cannot distinguish renewable from non-renewable resources, flow from stock pollution, externalised effects on utility from those on production, or domestic from foreign variables. Almost all dynamic economy-environmental models that address policy issues therefore include specific features; and inevitably the more features included, the more complex is the model. We include in the same model many features treated separately by Hartwick (1990), Maler (1991), Hamilton (1994), Vellinga and Withagen (1996), Sefton and Weale (1996) and many others, as follows.

The economy is generally open, competitive and small in relation to all world markets, though a closed economy can readily be considered as a special case. The economy's stock of a composite, depletable, natural resource is $S(t)$. This has both non-renewable and renewable characteristics, by being discovered at rate $D(t)$ and growing naturally at a stock-dependent rate $G(S)$. It is depleted at rate $R(t)$, so:

$$\dot{S} = D + G(S) - R;\quad S(t) \geq 0;\quad S(0) = S_0 > 0, \text{ given;} \qquad)$$

$$G_S\ (:= \partial G/\partial S) > 0 \text{ assumed for the economy's operating range.)} \qquad [2]$$[3]

Two more stocks in the domestic economy are productive capital $K(t)$ and abatement capital $K_a(t)$. They each increase at rates of investment $I(t)$ and $I_a(t)$, minus depreciation at a common rate $\delta > 0$:

$$\dot{K} = I - \delta K; \qquad K(t) \geq 0;\quad K(0) = K_0 > 0, \text{ given;} \qquad [3]$$

$$\dot{K}_a = I_a - \delta K_a; \qquad K_a(t) \geq 0;\quad K_a(0) = K_{a0} > 0, \text{ given.} \qquad [4]$$

Transient, polluting emissions $E(.)$ depend positively on domestic resource use (extraction $R(t)$ minus net exports $R_x(t)$), and negatively on abatement capital $K_a(t)$, abatement current expenditure $a(t)$, and time (exogenous technical progress in abatement).[4] Production $F(.)$ of a consumption-investment good depends positively on productive capital, domestic resource use, resource stock, and time (exogenous technical progress in production),

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[3] It would be more realistic to have many natural resources, and to distinguish non-renewable from renewable ones. However, with many renewable resources, their ecological interactions (where the stock S_i of one resource affects the growth G_j of another resource, so that the scalar G_S is replaced by a matrix $\{\partial G_j/\partial S_i\}$) cause big complications in calculating stock effects, which add little further insight.

[4] Contrast this with Bovenberg and Heijdra (1998), where the "dirty" variable creating emissions is the capital stock, and Jouvet et al (2000), where the dirty variable is output. In both papers, a tax on capital is desirable for environmental reasons.

and negatively on emissions.[5] The sum of production and net goods imports $M(t)$ is divided among consumption $C(t)$, productive and abatement investments, abatement current expenditure, and the combined costs $V(R,D,S,t)$ of discovering and extracting resources:

$$F[K,R-R_x,S,E(R-R_x,K_a,a,t),t] + M = C + I + I_a + a + V(R,D,S,t) \qquad [5]$$

$$F_K,\ F_R,\ F_S > 0,\ F_{KK},\ F_{RR},\ F_{SS} < 0,\ F_{KR} > 0,\ F_E < 0,\ F_{EE} < 0,\ F_t > 0 \qquad [6]$$

$$E_R > 0,\ E_K,\ E_a,\ E_t < 0;\quad V_R,\ V_D > 0,\ V_S,\ V_t < 0 \qquad [7]$$

The economy has a stock $K_f(t)$ of foreign capital (possibly negative, meaning debt) which earns a return at the exogenously varying world interest rate $r(t)$, while its net resource exports $R_x(t)$ are sold at exogenously varying world prices $Q^x(t)$. Foreign capital therefore grows as:

$$\dot{K}_f = r(t)K_f + Q^x(t).R_x - M; \qquad K_f(0) = K_{f0}, \text{ given} \qquad [8]$$

The case of a closed economy can easily be obtained, by eliminating net resource exports R_x and net goods imports M from the production equation [5], and deleting equation [8]. However, we do exclude the much more general and complex case of a large, open economy, which has power in one or more of world capital or resource markets. Prices r and Q^x would then depend on M and/or R^x, and strategic interactions with other economies would have to be considered.

Instantaneous utility $U(.)$ depends positively on consumption and resource stock, and negatively on emissions:

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[5] We could also model the effects of a cumulative pollutant on output or utility, but this adds little further insight to the emissions and resource stock effects included here.

$$\left.\begin{array}{l} U = U(C(t),S(t),E(t)); \\ U_C,\ U_S > 0,\ U_{CC},\ U_{SS} < 0,\ U_{CS} \geq 0,\ U_E < 0,\ U_{EE} < 0,\ \lim_{t\to\infty} U_C = 0. \end{array}\right) \quad [9]$$

Lastly, the economy chooses its control variables (of which an independent set is consumption C, abatement current expenditure a, abatement investment I_a, resource discoveries D, net imports M, resource extraction R, and net resource exports R_x), as if to maximise (intertemporal) *welfare* $W(0)$. This is the generalised present value (GPV) of utility, using a discount factor $\phi(t)$:

$$W(0) := \int_0^\infty \phi(t)U(C(t),S(t),E(t))dt; \quad \phi(t) > 0,\ \dot{\phi}(t) < 0,\ \phi(0) = 1. \qquad [10]$$[6]

Since any (intertemporally Pareto-) efficient development path can be found by maximising welfare in a competitive economy for some discount factor path (Takayama 1985, p188), the generality of ϕ in [10] includes any efficient path as possibly being optimal. All functional forms in [2]-[9] are assumed to be as smooth and convex as is needed for the existence of a unique and interior development path which maximises welfare. This will be called "optimal", or with the qualifications "socially optimal" or occasionally "GPV-optimal" if the context is not clear. If $\phi(t) = e^{-\rho t}$ with $\rho > 0$ constant, W will then be called just "present value" (PV), and the path maximising $W(0)$ is "PV-optimal". However, we also often consider a variable discount rate $\rho(t) = -\dot{\phi}(t)/\phi(t)$ corresponding to a general discount factor $\phi(t) = \exp[-\int_0^t \rho(z)dz]$. The context will make clear whether or not ρ is constant.

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[6] When the utility discount rate $-\dot{\phi}/\phi$ is not constant, it may be awkward to regard $W(0)$ as really "welfare", since for instance $W(t)$ (defined in Section 3.2) rises on the constant consumption path discovered by Solow (1974). So formally, $W(0)$ should just be regarded as what the economy maximises.

2.2 Environmental policy

As in most of the literature reviewed at the outset, we define the economy's environmental policy as just the time path of price incentives which the government must create, to induce private individuals in a decentralised equilibrium to follow the socially optimal path. Such intervention is needed because individuals are presumed to maximise $W(0)$ imperfectly, by ignoring (externalising) the "environmental" effects of their actions when making marginal choices of control variables. Such effects are here taken to be the partial derivatives of utility with respect to the resource stock, U_S, and emissions, U_E; of production with respect to emissions, F_E; of discovery and extraction costs with respect to stocks, V_S; and of resource growth with respect to the stock, G_S. The development path chosen as a result of such externalities is called "privately optimal". What is or is not externalised is part of the model's design, rather than required by logic. So for example, to model pollution by greenhouse gases, it would be the world rather than domestic resource stock (or instead, an explicit pollution stock) which affects utility or production. Or if renewable resources are privately owned, there would be no reason to suppose that G_S is externalised. Also, externalities from human-made resources like knowledge or education could be treated by similar techniques, but by convention such resources are not included in environmental policy.

Environmental policy is the set of incentives $\{\tau_i(t)\}$ (a tax when positive, or subsidy when negative) that the government can create to affect individuals' budget constraints, and hence make the privately optimal path coincide with the socially optimal path. Any net revenues from (or costs of) the incentive system are assumed to be refunded to (or taxed from) the representative consumer as lump sums which do not affect any marginal choices. To institute these incentive schemes credibly for all time would be difficult, but along with most optimal control modelling, we do not explore what constitutional innovations this might require in a democratic society. Neither do we consider the well-known political difficulties of collecting the lump sum taxes needed when the incentive scheme has net costs, or the administrative difficulties of managing tax rates that vary over time.

By using the device in Pemberton and Ulph (2001) of treating time t as a productive stock (with $\dot{t} = 1$), we can build the various ways in which production possibilities change exogenously (from technical progress or from changing terms of trade) into a standard optimal control format. The current value Hamiltonian for solving the social maximisation problem [10] subject to conditions [2]-[9] is then

$$\begin{aligned} H &= U + \Psi^K\dot{K} + \Psi^a\dot{K}_a + \Psi^f\dot{K}_f + \Psi^S\dot{S} + \Psi^t\,\dot{t} \\ &= U[C,S,E(R-R_x,K_a,a,t)] \\ &\quad + \Psi^K[F(K,R-R_x,S,E(R-R_x,K_a,a,t),t)+M-C-\delta K-a-I_a-V(R,D,S,t)] \\ &\quad + \Psi^a(I_a-\delta K_a) + \Psi^f(rK_f+Q^xR_x-M) + \Psi^S[D+G(S)-R] + \Psi^t \qquad [11] \end{aligned}$$

where Ψ^K, Ψ^a, Ψ^f, Ψ^S and Ψ^t are the respective co-state variables.

As for policy instruments, we do not consider quantity restrictions, and confine ourselves to proportional taxes with no thresholds or progressive changes in rates, unlike most income taxes in practice.[7] There are still many instruments available: taxes (or subsidies when rates are negative) on consumption (τ_C), productive capital (τ_K), productive investment (τ_I), abatement current spending (τ_a), abatement capital (τ_{Ka}), abatement investment (τ_{Ia}), resource discoveries (τ_D), resource extraction (τ_R), resource

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[7] Mohtadi (1996, Section 3.3) considers a quantity restriction which increases emissions abatement, but this is modelled as a "mandatory change in a parameter value", whereas here we have an explicit emission flow which can be taxed.

extraction costs (τ_V), net resource exports (τ_{Rx}), resource stock (τ_S), emissions (τ_E), net goods imports (τ_M) and foreign capital (τ_{Kf}). However, all of these instruments can be shown to be distortionary, apart from the emissions tax τ_E and the resource stock subsidy $-\tau_S$. Appendix 1 gives a partial proof of this, but for brevity it does not analyse every other tax, only taxes τ_C on consumption, τ_K on capital and τ_R on resource extraction.[8] These analyses will also prove useful later when discussing sustainability policy. The current value Hamiltonian [11] is then replaced by

$$\begin{aligned} H = {} & U(C,\bar{S},\bar{E}(R-R_x,K_a,a,t)) \\ & + \Psi^K\,[F(K,R-R_x,\bar{S},\bar{E}(R-R_x,K_a,a,t),t)+M-C-\delta K-a-I_a-V(R,D,\bar{S},t)] \\ & - \Psi^K\,[\tau_C C + \tau_R R + \tau_K K + \tau_S S + \tau_E E(R-R_x,K_a,a,t) - \Omega] \\ & + \Psi^a(I_a-\delta K_a) + \Psi^f(rK_f+Q^x R_x-M) + \Psi^S[D+G(\bar{S})-R] + \Psi^t. \end{aligned} \qquad [12]$$

Here Ω is the lump sum refund of all net tax revenues, and overbars show the environmental variables $\bar{S}$ and $\bar{E}$ that individuals ignore when making private, maximising choices. Appendix 1(c) shows that the required environmental policy at any time t is then the combination, with all quantities as measured on the socially optimal path, of:

$$\tau_C(t) = 0 \qquad [13]$$

$$\tau_K(t) = \tau_R(t) = 0 \qquad [14]$$

$$\tau_E(t) = -1/E_a(t) = -\,U_E(t)/U_C(t) - F_E(t) > 0 \qquad [15]$$

$$-\tau_S(t) = U_S(t)/U_C(t) + F_S(t) - V_S(t) + V_D(t)G_S(t) > 0 \qquad [16]$$

The intuition is simple. Tax τ_E internalises $-U_E/U_C$, the dollar-value amenity cost of emissions, and $-F_E$, the productivity cost of emissions. Subsidy $-\tau_S$ internalises the various benefits of the resource stock: U_S/U_C, the dollar-value

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[8] An investment tax/subsidy τ_I may be easier to administer in practice than a consumption tax τ_C or capital tax τ_K. However, the expression for its required form is more complex, and it does not add any extra insight, so we omit it.

amenity benefit; F_S, the productivity benefit; $-V_S$, the benefit from lower extraction costs; and $V_D G_S$, the benefit from the lower discoveries needed thanks to faster resource growth. Any taxes other than [15] and [16] will cause distortions that reduce welfare. One can thus be reasonably confident that, apart from the important exceptions of international interactions and cross-border externalities, "internalise all externalities at their source" is a useful general rule for dynamic environmental policy, at least in the first best case where time-varying incentives and lump sum taxes are available.

Note in passing that the U_E/U_C and U_S/U_C terms in [15] and [16] would typically be measured in dollars per tonne of emissions or resource stock. Data for them would have to come from the same, practically difficult non-market valuation exercises used for other methods of resource accounting and sustainability measurement, and the non-measurability of the utility concepts involved here does not add any extra difficulty.

3. Sustainability policy in the same economy

3.1 Sustainability policy versus social welfare maximisation

Before discussing the details of a sustainability policy, one must first recognise that whatever it is, its aim must be distinct from maximising social welfare as defined by the generalised present value $W(0)$ in [10] using the representative agent's discount factor $\phi(t)$. Sustainability policy is meaningless as a distinct concept in the neoclassical context unless one accepts that governments may rationally seek a policy which may not necessarily maximise $W(0)$.

How this apparent schizophrenia can exist, and how such a government could be democratically elected by agents who maximise present value (albeit imperfectly) in their private choices, are important questions in the economics of sustainability, which have not yet been fully resolved. One could just appeal to the idea in Marglin (1963, p98) that "...the Economic Man and the Citizen are for all intents and purposes two different individuals", and identify Economic Man as the private present-value-maximiser, and the Citizen as the supporter of sustainability as a public policy goal. More satisfying would be to develop the informal idea in Daly and Cobb (1989, p39) and Howarth and Norgaard (1993, p351) that sustainability is a partly-public good, because of the sexual intermixing of bequests across successive generations. We do not add any analysis here, and just accept that sustainability policy aims at some notion of intergenerational equity that is not captured by the welfare function $W(0)$.

3.2 Optimal and non-optimal sustainability policies

Whereas maximising welfare is a natural aim of environmental policy, the exact aim of sustainability policy is harder to define. Three possible aims which make sense in our neoclassical economy are:

(i) achieving constant utility forever (after Solow 1974 and Hartwick 1977);

(ii) avoiding any decline in utility, by ensuring that utility is always sustainable (after Pearce, Markandya and Barbier 1989 and Pezzey 1992);

(iii) avoiding any decline in welfare $W(t)$ measured from time t onwards, defined as $\int_t^\infty [\phi(s)/\phi(t)]U[C(s),S(s),E(s)]ds$, after Riley (1980).[9]

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[9] Non-declining wealth or non-declining aggregate capital, two further well-known alternatives suggested by Pearce et al, are best viewed in our context as (possibly

Any of these aims can in turn be related to the socially optimal and privately optimal paths of the economy in one of at least three ways:

(a) The sustainability aim is met on the privately optimal path, without any policy intervention. There is then no need for a sustainability policy as such.

(b) The aim is met on the socially optimal, but not on the privately optimal path. (The converse is logically possible, but unlikely.) Environmental policy will then achieve sustainability as an automatic side-effect.

(c) The aim is met on neither the privately optimal nor the socially optimal path. There is then a need for an explicit sustainability policy. This is the only case we consider hereafter.

Having assumed the need for an explicit sustainability policy, we further have to make clear if it is to be *optimal*. That is, should sustainability be achieved with minimum loss of welfare $W(0)$, which could be viewed as the best compromise of the aims of environmental policy and of sustainability policy? Let us consider this case first. If it exists, an optimal sustainable path is also Pareto efficient. From Takayama (1985) as already noted, this means that the optimal sustainable path maximises some measure of welfare, say $W^{\sigma}(0)$, based on a discount factor $\exp[-\int_0^t \sigma(z)dz]$, where $\sigma(t)$ is a "sustainable" discount rate. This rate is different from the private agent's $\rho(t)$, and reflects whatever degree of intergenerational equity (in a general form, constrained only by Pareto efficiency) that society collectively wants. The optimal sustainable path therefore satisfies first order conditions which are identical to those for the socially optimal path, save that $\sigma(t)$ replaces $\rho(t)$. Appendix 1(d) shows that one might hope the set of taxes and subsidies

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flawed) means, rather than fundamental ends, of sustainability policy.

$$\dot{\tau}_C(t)/[1+\tau_C(t)] = \sigma(t) - \rho(t) \qquad [17]$$

$$\tau_K(t) = \tau_R(t) = 0 \qquad [18]$$

$$\tau_E(t) = -\, U_E(t)/U_C(t) - F_E(t) = 1/E_a(t) > 0 \qquad [19]$$

$$-\tau_S(t) = U_S(t)/U_C(t) + F_S(t) - V_S(t) + V_D(t)G_S(t) > 0 \qquad [20]$$

would form the optimal sustainability policy, by inducing the representative agent to follow the path that maximises $W^{\sigma}(0)$.[10] However, Appendix 1(d) also shows that the consumption tax τ_C causes a distortion which forces the policy intervention path away from the W^{σ}-maximising path. So in general there is no exact optimal sustainability policy using the policy instruments considered here.[11]

If one abandons the requirement for sustainability to be optimal, then sustainability policies can generally be found. However, they may not be well defined, because by the argument above they cannot contain any maximising aim as well as a sustainability aim, since a path that achieved both aims would be Pareto-efficient, and thus unable to be achieved by policy intervention. So for example, a path of *maximum* constant utility cannot be reached by policy intervention, because it must be the socially optimal solution for some discount rate path $\sigma(t)$, and therefore unattainable.

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[10] The intuition behind [17] can readily be seen if both discount rates σ and ρ are constant, with $0 < \sigma < \rho$ because σ expresses stronger concern for future generations. $\tau_C(t)$ would then be a falling consumption tax or rising consumption subsidy, which gives an incentive to delay consumption and bring forward productive investment. It would change an individual's effective utility discount rate from ρ to σ, so this is almost a way of achieving Becker's direct manipulation of this rate.

[11] This minor impossibility result is absent from overlapping generations models, because the latter assume the feasibility of lump sum intergenerational transfers, which can shift consumption to the future without the use of a distorting marginal incentive.

However, Appendix 1(e) shows that constant utility can be achieved suboptimally by choosing a capital tax $\tau_K(t)$ and consumption tax $\tau_C(t)$ such that

$$\begin{aligned}&\tau_K + \dot{\tau}_C/(1+\tau_C)\\ &\qquad = F_K - \delta - \rho + [(\eta/C)(U_S\dot{S}+U_E\dot{E})+U_{CS}\dot{S}+U_{CE}\dot{E}]/U_C\\ &\qquad\qquad \text{where } \eta(C) := -U_{CC} / (U_C/C). \qquad [21]\end{aligned}$$

From the above, it is clear that environmental policies and sustainability policies interact. The environmental policy that would maximise welfare in the presence of a constant-utility sustainability policy such as [21], and that could not be Pareto-efficient for reasons already discussed, must generally be different from the welfare-maximising, environmental policy path defined by [13]-[16] which applies when there is no sustainability policy. This echoes a central result of Howarth and Norgaard (1992), that environmental valuations (here the sizes of the emissions tax τ_E and the resource stock subsidy $-\tau_S$) do not exist in a vacuum, independently of society's view on intergenerational equity. There will also be an interaction in the other direction. We conjecture that environmental policy to internalise a flow externality will have no effect on a sustainability policy, while environmental policy to internalise a stock externality will reduce the required strength of a sustainability policy. However, it is only in simple cases with specific functional forms, as illustrated next, that we can be clear about the sign and magnitude of such effects.

4. An example of the interaction of sustainability and environmental policies

We give here a simple example of an economy where asymptotic sustainability and environmental policies can be distinguished in both form and strength, and also interact. The economy is a variant on Stiglitz (1974), being closed, with no capital depreciation, a known, non-renewable resource, no emissions, and just one externality from the resource stock's role in a Cobb-Douglas production function with exogenous technical progress:

$$F(K,R,\bar{S},t) = K^{\alpha}R^{\gamma}S^{\chi}e^{\nu t} = \dot{K} + C; \quad 0 < \alpha, \gamma, \chi < 1; \;\; \alpha+\gamma+\chi \leq 1; \;\; \nu > 0 \qquad [22]$$

Utility is isoelastic and purely materialistic:

$$U(C) = C^{1-\eta}, \;\; 0 < \eta < 1. \qquad [23]$$

The utility discount rate ρ is a positive constant. To ensure respectively that the welfare integral converges and socially optimal utility declines asymptotically, we assume

$$(1-\alpha)\rho > (1-\eta)\nu \quad \text{and} \quad \rho > \nu/\gamma. \qquad [24]$$

Using results from Appendix 1 (denoted [A3], [A5], etc), we compute three asymptotic, balanced growth paths for this economy: the socially optimal path, the privately optimal path with sustainability policy only, and the privately optimal path with both sustainability and environmental policies. We denote $g_X := g\langle X\rangle := \lim_{t\to\infty}\dot{X}/X$ for any variable X. Since the resource is non-renewable, $\dot{S} = -R$, hence $g_S = g_R < 0$, $R = g_R S$. From [22], production, capital and consumption must all grow at the same asymptotic rate, so on all three paths:

$$g_F = \alpha g_K + (\gamma+\chi)g_R + \nu = g_C = g_K \;\Rightarrow\; (1-\alpha)g_C = (\gamma+\chi)g_R + \nu \qquad [25]$$

The socially optimal path

[A5] and [23] $\quad\Rightarrow\quad$ $-\eta g_C = \rho - F_K$ [26]

[A3], [A5], [A7] and [A1]

$$\Rightarrow \dot{F}_R/F_R = g<\gamma F/R> = F_K - F_S/F_R$$

$$\Rightarrow g_C - g_R = F_K - (\chi F/S)/(\gamma F/R)$$

$$\Rightarrow g_C = F_K + (1+\chi/\gamma)g_R \qquad [27]$$

[26], [27] and [25]

$$\Rightarrow (1-\eta)g_C = \rho + (1+\chi/\gamma)g_R = \rho + [(1-\alpha)g_C-\nu]/\gamma$$

$$\Rightarrow [(1-\eta)\gamma-(1-\alpha)]g_C = \rho\gamma - \nu$$

$$\Rightarrow g_C = (\nu-\rho\gamma) / [(1-\alpha)-(1-\eta)\gamma] < 0 \text{ from [24]} \qquad [28]$$

The privately optimal path with sustainability policy (τ_C) only

[25] and [26] still hold, while F_S is ignored so [27] is replaced by

$$\Rightarrow g_C = F_K + g_R \qquad [29]$$

[A15] and [23] $\Rightarrow -\eta g_C - \dot{\tau}_C/(1+\tau_C) = \rho - F_K$ [30]

[27], [30] and [25]

$$\Rightarrow (1-\eta)g_C - \dot{\tau}_C/(1+\tau_C) = \rho + [(1-\alpha)g_C-\nu]/(\gamma+\chi)$$

$$\Rightarrow [(1-\eta)(\gamma+\chi)-(1-\alpha)]g_C = [\rho+\dot{\tau}_C/(1+\tau_C)](\gamma+\chi) - \nu$$

$$\Rightarrow g_C = [\nu-(\rho+\dot{\tau}_C/(1+\tau_C))(\gamma+\chi)] / [(1-\alpha)-(1-\eta)(\gamma+\chi)] \qquad [31]$$

The privately optimal path with sustainability policy (τ_C) and environmental policy ($-\tau_S$)

From [A19], the environmental policy is a resource stock subsidy $-\tau_S = F_S = \chi F/S$ (not constant), which causes [27] to be reinstated, while [30] still holds, so

[27], [30] and [25]

$$\Rightarrow (1-\eta)g_C - \dot{\tau}_C/(1+\tau_C) = \rho + [(1-\alpha)g_C-\nu]/\gamma$$

$$\Rightarrow [(1-\eta)\gamma-(1-\alpha)]g_C = [\rho+\dot{\tau}_C/(1+\tau_C)]\gamma - \nu$$

$$\Rightarrow g_C = [\nu-(\rho+\dot{\tau}_C/(1+\tau_C))\gamma] / [(1-\alpha)-(1-\eta)\gamma] \qquad [32]$$

We can then see the interaction between the two policies: *environmental policy makes sustainability policy easier*. The required strength of sustainability policy (to make consumption constant, i.e. $g_C = 0$) is lower if environmental policy is already in place: $-\dot{\tau}_C/(1+\tau_C) = \rho - \nu/\gamma$ from [32], instead of the larger $-\dot{\tau}_C/(1+\tau_C) = \rho - \nu/(\gamma+\chi)$ from [31].[12] The amount of difference made by environmental policy is related to χ, the strength of the stock externality in the production function.

However, we cannot show analytically that sustainability policy reduces the required strength of environmental policy $\chi F/S$, because the latter changes over time. One would suspect, though, that such an effect could be demonstrated numerically, thus matching the finding in Howarth and Norgaard's (1992) overlapping generations model.

5. Two extra results in more specialised economies

5.1 The powerlessness of resource incentives to achieve constant utility

We consider here a special case, where the economy is closed, there is no resource discovery or renewal ($D = G = 0$, so $\dot{S} = -R$), the discount rate ρ is a positive constant, there is no technical progress in production ($F_t = 0$), and there is no amenity effect of emissions ($U_E = 0$). Since the resource is finite and non-renewable, the resource depletion rate R be asymptotically zero. With $F = F(K,R)$ only and $F_{KR} > 0$, it is then reasonable to make this:

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[12] When $\nu/(\gamma+\chi) < \rho \leq \nu/\gamma$, unlike in [24], no asymptotic sustainability policy is required if environmental policy is enacted, but sustainability policy is still required if it is not. When $\rho \leq \nu/(\gamma+\chi)$, no sustainability policy is required in either case. In all cases, one cannot tell whether utility rises or falls in the (potentially long) period before development comes close enough to the asymptotic, balanced growth path.

Assumption

On the path [A11]-[A20], $\lim_{t\to\infty} F_K =: \xi,\ \ 0 \le \xi < \rho+\delta.$ [33]

That is, no matter what policy instruments are used, the privately optimal return on capital F_K (the competitive interest rate) eventually falls below $\rho+\delta$. (Proving the exact conditions under which [33] holds is complex even for simple economies, as shown by Pezzey and Withagen (1998).)

We can then establish:

Proposition: Under the above conditions, no matter what $\tau_R(t)$ and $\tau_S(t)$ schedules the government creates as policy interventions, if these are the only interventions, then $\lim_{t\to\infty}\dot{U} < 0$. In other words, *resource depletion and resource stock incentives* $\tau_R(t)$ *and* $\tau_S(t)$ *are powerless to prevent unsustainability* in the form asymptotically falling utility.

Proof: Appendix 1(e) shows that under the above conditions, utility on the policy path with intervention changes as

$$\dot{U} = \{[F_K-\rho-\delta-\tau_K-\dot{\tau}_C/(1+\tau_C)]U_C - R(U_{CS}+\eta U_S/C)\}\ C\ /\ \eta. \qquad [34]$$

Resource incentives τ_R and τ_S do not appear in [34], and since these are the only policy interventions, $\tau_K = \tau_C = 0$. [33], with $U_C > 0$ from [9], then means that $\lim_{t\to\infty}[F_K-\rho-\delta-\tau_K-\dot{\tau}_C/(1+\tau_C)]U_C < 0$. Also $U_{CS} \ge 0$ and $U_S > 0$ from [9], so the stock amenity term $-R(U_{CS}+\eta U_S/C) < 0$. So both terms in [34] < 0 asymptotically, hence $\lim_{t\to\infty}\dot{U} < 0$. Q.E.D.

Of course, resource taxes and subsidies will have some effect on $\dot{U}$, via the return on capital F_K, as seen from the Hotelling rule for the privately optimal path with intervention which is calculated in Appendix 1(b):

$$F_K - \delta - \tau_K = [(d/dt)(F_R - V_R - \tau_R - \tau_E E_R) - \tau_S] \,/\, (F_R - V_R - \tau_R - \tau_E E_R) \qquad [35]$$

However, given the assumption in [33], no sustained (i.e. asymptotic) effect of τ_R and/or τ_S on F_K is possible. Intuitively, τ_R and τ_S can raise $F_K(K,R)$ by giving an incentive to increase the resource flow R, but such a non-vanishing R can be sustained for only a finite time by a finite stock S_0. [34] shows that the only way to raise $\dot{U}$ by a sustained, finite amount is to use a combined instrument such that eventually $\tau_K + \dot{\tau}_C/(1+\tau_C) < 0$, i.e. a capital subsidy and/or a falling consumption tax. The effect of this is to induce people to invest more, when they would normally look for a return of at least $\rho+\delta$ on their investment, but the marginal return on capital investment is dwindling towards zero. The sustainability policy eventually involves the need for lump sum taxes to pay for subsidies; for even if only the consumption tax τ_C is used, Appendix 2 shows that it must ultimately be a 100% subsidy:

$$\lim_{t\to\infty} \dot{\tau}_C/(1+\tau_C) \le -(\rho+\delta-\xi) < 0 \quad \Rightarrow \quad \lim_{t\to\infty} \tau_C = -1 \qquad [36]^{13}$$

5.2 The separation of domestic allocation and sustainability policy in a small open economy with limited environmental effects

Here we consider a special case of the economy which is still small and open, but where there are no environmental amenity effects ($U_E = U_S = 0$), no environmental resource stock effects ($F_S = V_S = 0$), leaving only $F_E < 0$ as a possible environmental effect. As shown earlier, the only policy tools that need to be considered are an emissions tax $\tau_E(t)$, a stock subsidy $-\tau_S(t)$

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[13] As an example, one can show that a consumption tax/subsidy path $\tau_C(t) = [1+((1/\alpha)-1)\bar{C}/K_0]^{\alpha/(1-\alpha)}e^{-\rho t}-1$ converts the PV-optimal path of the capital-resource economy with discount rate ρ and $F(K,R) = K^\alpha R^{1-\alpha} = \dot{K} + C$, to the Solow (1974) path of maximum constant consumption, $C(t) = \bar{C} := \alpha\{K_0^{2\alpha-1}[(2\alpha-1)S_0]^{1-\alpha}\}^{1/\alpha}$.

and a consumption tax $\tau_C(t)$ (including a capital incentive τ_K would add no extra insight). In such a case, Appendix 3 shows that there is a separation between domestic production decisions and environmental policy on the one hand, and consumption, trade, foreign investment and sustainability policy on the other hand. Specifically, because the interest rate $r(t)$ is exogenous, consumption (and hence utility $U(C)$) is completely determined by the sustainability policy (τ_C). Conversely, productive capital K, resource extraction R, net resource exports R_x, resource discoveries D, abatement capital K_a and abatement current spending a are completely unaffected by sustainability policy, being affected only by environmental policy (τ_E and $-\tau_S$).[14] What does also change as a result of sustainability policy is a country's net imports M, and the rate at which it acquires foreign capital K_f.

Suppose now that the privately optimal path, with or without environmental policy but with no sustainability policy intervention, would follow a path where the economy's natural resources are completely depleted, and development is unsustainable. A sustainability policy will then make no difference to how resources and production are managed; its *only* result is less consumption and more saving, with all the saving being invested in foreign capital. This is essentially another version of Fisher's (1930/1954, p271) "separation theorem", where the separation of consumption and saving decisions from depletion and production decisions follows from the exogeneity of the interest rate and resource prices.

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[14] However, if sustainability policy is present, then as previously argued, this distorts the consumption decision so that environmental policy cannot achieve a Pareto-efficient path. There may then be no unique aim for environmental policy.

This scenario, of achieving sustainability by stripping domestic resources and investing the proceeds abroad, would flatly contradict the idea that sustainability requires *resource* policies. This idea, based on assumed non-substitutability of humanmade capital for natural resources, and promoted by Pearce (1988), Daly (1990) and many other authors since as one of the cardinal rules of sustainability, is that domestic natural resources must be conserved in some way. However, our derivation of a "strip resources and invest abroad" policy is not intended to recommend it. Such a policy would be wise only in the unlikely event that resources have only private productive value, and no intrinsic, amenity or publicly productive value; that capital will always be substitutable for resources in domestic production; that all this is known with certainty; and that few other countries are planning to adopt the same policy, so that no fallacy of composition occurs. If all countries followed the policy, there would obviously be no abroad left to conserve natural resources and accept incoming investments.

6. Conclusions

Using a fairly general, representative agent, neoclassical model of a dynamic economy which is small and open, or closed, we have shown how environmental policy and sustainability policy, terms used interchangeably in much policy debate, can be defined theoretically in quite distinct ways. They can be distinct in their aims, and in the instruments needed to achieve these aims. Environmental policy was defined as dynamic intervention to maximise intertemporal social welfare, and it needs to internalise the social values of "environmental" stocks and flows that agents ignore when they maximise private welfare. Sufficient instruments to achieve this are incentives (taxes or subsidies, with any costs or revenues neutralised by lump sum transfers) directly on the environmental stocks and flows, equal to their social values in equilibrium. Any incentives applied directly to intermediate variables, like resource depletion or emissions abatement, will be distortionary. This conclusion is unaffected by resource discovery and extraction costs, trade in goods and resources, abatement of emissions by current or capital spending, and exogenous technical progress, all of which were included in our model.

By contrast, sustainability policy aims to achieve some improvement in intergenerational equity, whether a general shift to a lower path of the utility discount rate over time, or a specific aim to make utility forever constant, non-declining or sustainable. We assumed, without giving any formal analysis, that such equity could not be represented in the original social welfare function based on individual preferences, and that people may support governments that try to achieve intergenerational equity with a sustainability policy which prevents welfare maximisation. In the absence of lump sum intergenerational transfers or directly manipulable interest rates, sustainability policy was showed to use consumption or capital incentives, but not resource incentives, to affect consumption and savings choices over time. However, this cannot achieve optimal sustainability (sustainability that maximises some redefined measure of welfare), because optimality requires an undistorted choice between consumption and investment. But policy can achieve a sustainability objective on its own, such as constant utility.

Sustainability policy will clearly interact with environmental policy, but it is hard to say how in general. We conjectured that environmental policy that internalises the cost of cumulative pollution or resource degradation will somehow improve sustainability. We showed analytically, in an asymptotic, Cobb-Douglas, capital-resource economy, how the presence of environmental policy lowers the required strength of sustainability policy. The fact that sustainability policy requires incentives on consumption or capital is also illustrated by results in more restricted economies. If the economy is closed with a constant discount rate and strictly non-renewable resources, then the net return to capital is likely to fall below the discount rate, which means that resource incentives are ultimately powerless to achieve sustainability. Only consumption or capital incentives, ultimately subsidies, will suffice. If the economy is small and open with limited environmental externalities, then not only does sustainability policy not use resource incentives, but also it has absolutely no effect on resource management or domestic production. It is then theoretically possible for a small economy acting in isolation to achieve sustainable development while stripping its domestic natural resources down to zero, as long as its consumption is restrained and enough is invested in foreign capital stocks.

These results do not suggest that in a more realistic policy context, sustainability and environmental policies can and should be considered in separate, watertight compartments. The analysis is not at all complete, as education and knowledge accumulation, international market power and strategic interactions, cross-border environmental effects and second-best policy instruments have all been ignored, and remain as obvious topics for further work. However, our analysis does suggest a rather different focus than has appeared to date in most neoclassical economic literature on sustainability, which stresses definition, justification, measurement and accounting rather than policy intervention. The focus is also different than most ecological economic literature, which almost exclusively stresses action to protect environmental resources. To be complete, sustainability analysis needs to give more attention to policy intervention that will encourage adequate saving and investment.

Appendix 1

We calculate the socially optimal path and the privately optimal path with policy interventions, and thus the interventions which make the two coincide.

(a) The socially optimal path

From the Hamiltonian in [11], an interior solution to the problem of maximising [10] subject to [2]-[9] satisfies the first order conditions:

$\partial H/\partial C = 0 = U_C - \Psi^K \quad \Rightarrow \quad \Psi^K = U_C$ [A1]

$\partial H/\partial a = 0 = U_E E_a + \Psi^K(F_E E_a - 1)$

$\Rightarrow \quad (U_E/U_C + F_E)E_a = 1$ [A2]

$\partial H/\partial I_a = 0 = -\Psi^K + \Psi^a \quad \Rightarrow \quad \Psi^a = U_C$

$\partial H/\partial D = 0 = -\Psi^K V_D + \Psi^S$

$\Rightarrow \quad \Psi^S/U_C = V_D,\ \dot{\Psi}^S/\Psi^S = \dot{U}_C/U_C + \dot{V}_D/V_D$

$\partial H/\partial M = 0 = \Psi^K - \Psi^f \quad \Rightarrow \quad \Psi^f = \Psi^K = U_C$

$\partial H/\partial R = 0 = U_E E_R + \Psi^K(F_R + F_E E_R - V_R) - \Psi^S$

$\Rightarrow \quad (U_E/U_C + F_E)E_R + F_R - V_R = V_D$ [A3]

$\partial H/\partial R_x = 0 = -U_E E_R - \Psi^K(F_R + F_E E_R) + \Psi^f Q^x$

$\Rightarrow \quad Q^x = (U_E/U_C + F_E)E_R + F_R$ [A4]

$\partial H/\partial K = \rho\Psi^K - \dot{\Psi}^K = \Psi^K(F_K - \delta)$

$\Rightarrow \quad \dot{U}_C/U_C = \rho - F_K + \delta$ [A5]

$\partial H/\partial K_a = \rho\Psi^a - \dot{\Psi}^a = U_E E_K + \Psi^K F_E E_K - \Psi^a\delta$

$\Rightarrow \quad \dot{U}_C/U_C = \rho - (U_E/U_C + F_E)E_K + \delta$ [A6]

$\partial H/\partial K_f = \rho\Psi^f - \dot{\Psi}^f = \Psi^f r$

$\Rightarrow \quad \dot{U}_C/U_C = \rho - r$ [A7]

$\partial H/\partial S = \rho\Psi^S - \dot{\Psi}^S = U_S + \Psi^K(F_S - V_S) + \Psi^S G_S$

$\Rightarrow \quad \dot{\Psi}^S/\Psi^S = \rho - [U_S + U_C(F_S - V_S)]/U_C V_D - G_S$

[A8]

$$\Rightarrow \quad r - \dot{V}_D/V_D = (U_S/U_C+F_S-V_S)/V_D + G_S \tag{A9}$$

$$\partial H/\partial t = \rho\Psi^t - \dot{\Psi}^t = U_E E_t + \Psi^K(F_E E_t+F_t-V_t) \tag{A10}$$

(b) The privately optimal path with policy intervention

From the Hamiltonian [12], the first order conditions satisfied by an interior solution of the privately optimal path with intervention are:

$$\partial H/\partial C = 0 = U_C - \Psi^K(1+\tau_C)$$

$$\Rightarrow \quad \Psi^K = U_C/(1+\tau_C) \tag{A11}$$

$$\partial H/\partial a = 0 = -\Psi^K(1+\tau_E E_a)$$

$$\Rightarrow \quad \tau_E = -1/E_a \tag{A12}$$

$$\partial H/\partial I_a = 0 = -\Psi^K + \Psi^a$$

$$\Rightarrow \quad \Psi^a = U_C/(1+\tau_C)$$

$$\partial H/\partial D = 0 = -\Psi^K V_D + \Psi^S$$

$$\Rightarrow \quad \Psi^S/\Psi^K = \Psi^S(1+\tau_C)/U_C = V_D$$

$$\Rightarrow \quad \dot{\Psi}^S/\Psi^S + \dot{\tau}_C/(1+\tau_C) = \dot{U}_C/U_C + \dot{V}_D/V_D$$

$$\partial H/\partial M = 0 = \Psi^K - \Psi^f$$

$$\Rightarrow \quad \Psi^f = \Psi^K = U_C/(1+\tau_C)$$

$$\partial H/\partial R = 0 = \Psi^K(F_R-V_R-\tau_R-\tau_E E_R) - \Psi^S$$

$$\Rightarrow \quad F_R - V_R - \tau_R - \tau_E E_R = V_D \tag{A13}$$

$$\partial H/\partial R_x = 0 = -\Psi^K(F_R-\tau_E E_R) + \Psi^f Q^x$$

$$\Rightarrow \quad Q^x = F_R - \tau_E E_R \tag{A14}$$

$$\partial H/\partial K = \rho\Psi^K - \dot{\Psi}^K = \Psi^K(F_K-\delta-\tau_K)$$

$$\Rightarrow \quad \dot{U}_C/U_C - \dot{\tau}_C/(1+\tau_C) = \rho - F_K + \delta + \tau_K \tag{A15}$$

$$\partial H/\partial K_a = \rho\Psi^a - \dot{\Psi}^a = -\Psi^K\tau_E E_K - \Psi^a\delta$$

$$\Rightarrow \quad \dot{\Psi}^a/\Psi^a = \rho + \delta + (\Psi^K/\Psi^a)\tau_E E_K$$

$$\Rightarrow \quad \dot{U}_C/U_C - \dot{\tau}_C/(1+\tau_C) = \rho + \tau_E E_K + \delta \tag{A16}$$

$$\partial H/\partial K_f = \rho\Psi^f - \dot{\Psi}^f = \Psi^f r$$

$$\Rightarrow \quad \dot{\Psi}^f/\Psi^f = \rho - r$$

$$\Rightarrow \quad \dot{U}_C/U_C - \dot{\tau}_C/(1+\tau_C) = \rho - r \tag*{[A17]}$$

$$\partial H/\partial S = \rho\Psi^S - \dot{\Psi}^S = -\,\Psi^K\tau_S$$

$$\begin{aligned} \Rightarrow \quad \dot{\Psi}^S/\Psi^S &= \rho + \tau_S/V_D \\ &= \dot{U}_C/U_C + \dot{V}_D/V_D - \dot{\tau}_C/(1+\tau_C) \\ &= \rho - r + \dot{V}_D/V_D \end{aligned} \tag*{[A18]}$$

$$\Rightarrow \quad r - \dot{V}_D/V_D = -\tau_S/V_D. \tag*{[A19]}$$

$$\partial H/\partial t = \rho\Psi^t - \dot{\Psi}^t = \Psi^K(F_t - V_t - \tau_E E_t) \tag*{[A20]}$$

Using [A15] and [A17] to derive

$$r = F_K - \delta - \tau_K, \tag*{[A21]}$$

[A21] and [A13] can then transform [A19] into

$$\begin{aligned} &F_K - \delta - \tau_K \\ &\quad = [(d/dt)(F_R - V_R - \tau_R - \tau_E E_R) - \tau_S] \,/\, (F_R - V_R - \tau_R - \tau_E E_R), \end{aligned} \tag*{[35]}$$

which is the Hotelling rule for the privately optimal path.

(c) Environmental policy

The tax paths that constitute environmental policy are those that make identical the corresponding pairs of equations in the socially optimal and privately-optimal-with-policy solutions:

Comparing [A1] and [A11]

$$\Rightarrow \quad \tau_C = 0, \text{ and hence } \Psi^K = U_C. \tag*{[A22]}$$

Comparing [A2] and [A12]

$$\Rightarrow \quad \tau_E = -1/E_A = -(U_E/U_C + F_E). \tag*{[A23]}$$

Comparing the pairs of equations from $\partial H/\partial I_a$, $\partial H/\partial D$ and $\partial H/\partial M$, and [A7] and [A17], confirms [A22].

Comparing [A3] and [A13]

$$\Rightarrow \quad \tau_R = 0. \qquad [A24]$$

Comparing [A4] and [A14], [A6] and [A16], or [A10] and [A20], confirms [A23].

Comparing [A5] and [A15]

$$\Rightarrow \quad \tau_K = \dot{\tau}_C/(1+\tau_C) = 0. \qquad [A25]$$

Comparing [A9] and [A18]

$$\Rightarrow \quad -\tau_S = U_S/U_C + F_S - V_S + V_D G_S. \qquad [A26]$$

(d) An optimal sustainability policy?

Optimal sustainability aims to achieve the socially optimal path that would result from maximising welfare $W^\sigma(0)$ defined with a "sustainable" discount rate $\sigma(t)$, rather than with the rate $\rho(t)$ that the representative agent uses in private optimisation. The first order conditions satisfied by this W^σ-maximising-path are as in [A1]-[A10], except with ρ replaced by σ in equations [A5]-[A7], which together become:

$$\dot{U}_C/U_C = \sigma - F_K + \delta = \sigma - (U_E/U_C+F_E)E_K + \delta = \sigma - r \qquad [A27]$$

Inserting policies [17]-[20] into the conditions [A12]-[A20] for the privately optimal path with policy intervention, can be seen by inspection to make them identical with the equivalents on the W^σ-path. However, the need for a non-zero consumption tax τ_C, so that [A15]-[A17] can match [A27], means that $\Psi^K = U_C/(1+\tau_C)$ in [A11] can never match the need for $\Psi^K = U_C$ in the equivalent of [A1]. So no set of policies from those under consideration can make the privately optimal path with intervention the same as the optimal sustainable path.

(e) Sustainability-only policy

We first calculate an expression for the rate of change of utility:

$$\dot{U}_C = U_{CC}\dot{C} + U_{CS}\dot{S} + U_{CE}\dot{E}$$

$$\Rightarrow\ (U_{CC}/U_C)\dot{C} + (U_{CS}\dot{S}+U_{CE}\dot{E})/U_C = \dot{U}_C/U_C$$

$$= \dot{\tau}_C/(1+\tau_C) + \rho - F_K + \delta + \tau_K \text{ from [A15]} \qquad \text{[A28]}$$

Using $U_{CC}/U_C = -\eta(C)/C$, this means that

$$(-\eta/C)\dot{C} = \dot{\tau}_C/(1+\tau_C) + \rho - F_K + \delta + \tau_K - (U_{CS}\dot{S}+U_{CE}\dot{E})/U_C$$

$$\Rightarrow \dot{C} = [\, F_K - \rho - \delta - \tau_K - \dot{\tau}_C/(1+\tau_C) + (U_{CS}\dot{S}+U_{CE}\dot{E})/U_C \,]\, C/\eta \qquad \text{[A29]}$$

$$\Rightarrow \dot{U} = U_C\dot{C} + U_S\dot{S} + U_E\dot{E}$$

$$= \{[F_K-\rho-\delta-\tau_K-\dot{\tau}_C/(1+\tau_C)]U_C + U_{CS}\dot{S} + U_{CE}\dot{E}\}C/\eta + U_S\dot{S} + U_E\dot{E} \qquad \text{[A30]}$$

The capital tax $\tau_K(t)$ and consumption tax $\tau_C(t)$ that are needed to make the privately optimal path with intervention have constant utility are thus

$$\tau_K + \dot{\tau}_C/(1+\tau_C)$$

$$= F_K - \delta - \rho + [(\eta/C)(U_S\dot{S}+U_E\dot{E})+U_{CS}\dot{S}+U_{CE}\dot{E}]/U_C. \qquad \text{which is [21]}$$

For later reference, when $U_E = 0$ and $\dot{S} = -R$, [A30] reduces to

$$\dot{U} = \{[F_K-\rho-\delta-\tau_K-\dot{\tau}_C/(1+\tau_C)]U_C - R(U_{CS}+\eta U_S/C)\}C/\eta \qquad \text{which is [34]}$$

Appendix 2

To prove: If $\dot{\tau}_C/(1+\tau_C) < 0$ and bounded away from zero after some time, then $\lim_{t\to\infty}\tau_C = -1$. *Proof:* the subsidy rate $\tau_C > -1$, or else an individual's desired consumption would be unbounded. Hence $\dot{\tau}_C < 0$, to make $-\dot{\tau}_C/(1+\tau_C) > 0$. So $\lim_{t\to\infty}\tau_C = -1+z$ for some finite $z \geq 0$, and $\lim_{t\to\infty}\dot{\tau}_C = 0$. But then $\lim_{t\to\infty}[-\dot{\tau}_C/(1+\tau_C)] = 0/z$, and $\lim_{t\to\infty}[-\dot{\tau}_C/(1+\tau_C)] > 0$ by assumption. Hence $z = 0$.

Appendix 3

If $U_E = U_S = F_S = V_S = 0$, and the only policy instruments are τ_E, τ_S and τ_C, then [A12]-[A19], the first order conditions of the privately optimal path with policy intervention can be rewritten (using [A36] in the derivation of [A34] and [A35]) respectively as:

$$\tau_E(t) = 1/E_a(R\text{-}R_x,K_a,a,t) \qquad \text{[A31]}$$

$$F_R(K,R\text{-}R_x,E(R\text{-}R_x,K_a,a,t),t) - V_R(R,D,t) - \tau_E(t)E_R(R\text{-}R_x,K_a,a,t) = V_D(R,D,t) \qquad \text{[A32]}$$

$$Q^x(t) = F_R(K,R\text{-}R_x,E(R\text{-}R_x,K_a,a,t),t) - \tau_E(t)E_R(R\text{-}R_x,K_a,a,t) \qquad \text{[A33]}$$

$$r(t) = F_K(K,R\text{-}R_x,E(R\text{-}R_x,K_a,a,t),t) - \delta \qquad \text{[A34]}$$

$$r(t) = -\tau_E(t)E_K(R\text{-}R_x,K_a,a,t) - \delta \qquad \text{[A35]}$$

$$\dot{U}_C(C)/U_C(C) = \rho(t) - r(t) + \dot{\tau}_C/(1+\tau_C) \qquad \text{[A36]}$$

$$r(t) - \dot{V}_D(R,D,t)/V_D(R,D,t) = -\tau_S(t)/V_D(R,D,t) \qquad \text{[A37]}$$

Equations [A31]-[A35] and [A37] are 6 equations which in principle determine the 6 unknowns K, R, R_x, D, K_a and a. Resource discovery and extraction costs $V(.)$, domestic production $F(.)$ and emissions $E(.)$ are then fully determined, independent of sustainability policy in the form of $\tau_C(t)$. $\tau_C(t)$ completely determines C and U via [A36]; and if we choose a sustainability policy $-\dot{\tau}_C/(1+\tau_C) = \rho - r$, we achieve constant utility, $\dot{U} = 0$. The only other variables apart from C affected by sustainability policy τ_C are

net imports $M = C + \delta K + V(R,D,t) - F(K,R\text{-}R_x,t)$, and hence

foreign capital K_f via $\dot{K}_f = rK_f + Q^xR_x - M$.

So sustainability policy affects only C, M and K_f, not K, R, R_x, D, K_a or a.

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